Online Appendix to

Decomposing Monetary Policy Surprises: Shock, Information, and Policy Rule Revision

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Abstract

This online appendix contains model derivations, details on the data used, and additional results for the paper 'Decomposing Monetary Policy Surprises: Shock, Information, and Policy Rule Revision.'

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Contents

A Modelling policy shocks and shocks to the rule

A.1 Nonlinear and linear policy rules

Let us consider a generic nonlinear Taylor rule, following Woodford (2003):

$$R_t = \phi\left(\frac{\Pi_t}{\Pi_t^*}; \nu_t\right), \tag{1}$$

where R_t is the gross nominal interest rate, and the function ϕ indicates the rule used by the central bank to set its policy rate. $\phi(\cdot; \nu_t)$ is a bounded-below, non-decreasing function for each possible value of the shifter ν_t, while $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate and Π_t^* is a, possibly, time-varying target rate.[1] ν_t captures shifts in the central bank's rule – i.e. variations in policy, or in its implementation –, distinct from changes in the inflation target itself.

A.1.1 A standard linearisation

The standard log-linearisation of Eq. (1) is obtained by first defining $r_t \equiv \log R_t$, $\pi_t \equiv \log \Pi_t$, and $\pi_t^* \equiv \log \Pi_t^*$, and then considering a first-order Taylor expansion of Eq. (1) at the point $(\Pi_t = \Pi_t^* = 1; \nu_t = 0)$. This corresponds to a Taylor expansion around a zero-inflation steady state.

We can then write

$$\begin{aligned} R_t =& \phi\left(\frac{\Pi_t}{\Pi_t^*}; \nu_t\right)\bigg|_{\Pi_t=1;\Pi_t^*=1;\nu_t=0} + \frac{\partial \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)}\bigg|_{\Pi_t=1;\Pi_t^*=1;\nu_t=0}\left(\frac{\Pi_t}{\Pi_t^*} - 1\right) \\ &+ \frac{\partial \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial \nu_t}\bigg|_{\frac{\Pi_t}{\Pi_t^*}=1;\nu_t=0} \nu_t + \mathcal{O}(2), \end{aligned}$$

[1] In general, $\phi(\cdot, \cdot)$ is a function capturing the nonlinear behaviour of the policy away from the steady state where a small perturbation approximation is not accurate, or around the zero or effective lower bound where its non-negative (or bounded-below) nature is visible.

and by employing the definitions above, we get

$$e^{r_t} = \phi_{0,t} + \left.\frac{\partial\phi\left(\Pi_t/\Pi_t^*;\nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)}\right|_{\Pi_t=1;\Pi_t^*=1;\nu_t=0}\left(e^{\pi_t-\pi^*}-1\right) + \left.\frac{\partial\phi\left(\Pi_t/\Pi_t^*;\nu_t\right)}{\partial\nu_t}\right|_{\Pi_t=1;\Pi_t^*=1;\nu_t=0}\nu_t + \mathcal{O}(2),$$

We can now proceed with a standard expansion to the first-order to obtain

$$1 + r_t = \phi_{0,t} + \left.\frac{\partial\phi\left(\Pi_t/\Pi_t^*;\nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)}\right|_{\Pi_t=1;\Pi_t^*=1;\nu_t=0}\left(\pi_t-\pi_t^*\right) + \left.\frac{\partial\phi\left(\Pi_t/\Pi_t^*;\nu_t\right)}{\partial\nu_t}\right|_{\Pi_t=1;\Pi_t^*=1;\nu_t=0}\nu_t + \mathcal{O}(2),$$

and by defining the following parameters

$$\begin{aligned}
r^* &\equiv \phi_{0,t} - 1 \equiv \left.\phi\left(\frac{\Pi_t}{\Pi_t^*};\nu_t\right)\right|_{\Pi_t=1;\Pi_t^*=1;\nu_t=0} - 1,\\
\phi &\equiv \left.\frac{\partial\phi\left(\Pi_t/\Pi_t^*;\nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)}\right|_{\Pi_t=1;\Pi_t^*=1;\nu_t=0},\\
\sigma_{mp} &= \left.\frac{\partial\phi\left(\Pi_t/\Pi_t^*;\nu_t\right)}{\partial\nu_t}\right|_{\Pi_t=1;\Pi_t^*=1;\nu_t=0},
\end{aligned}$$

we obtain the standard linear Taylor rule

$$r_t = r^* + \phi(\pi_t - \pi_t^*) + \sigma_{mp}\nu_t^{mp} + \mathcal{O}(2). \tag{2}$$

A few key observations are worth noting. First, ν, which captures shifts in the central bank's rule – i.e. the central bank becoming more hawkish or more dovish – in the standard approach is the source of monetary policy shocks. Second a shock to the target, depending on whether it is permanent or temporary, would either affect r^*, or manifest as a monetary policy shock. To obtain a Taylor rule with a time-varying response parameter to inflation, we need to take a slightly different expansion.

A.1.2 A linearised Taylor rule with a time varying parameter

Let us now consider a first-order log-linear approximation at a different value of the policy shifter, i.e. $\nu_t = \bar{\nu}_t$. In particular, we want to consider this parameter as

$$\nu_t = \bar{\nu}_t + \nu_t^{mp},$$

where $\bar{\nu}_t$ is persistent component – similar to a random walk, to fix ideas –, while ν_t^{mp} is a transitory component.

As was done before, let us define $r_t \equiv \log R_t$, $\pi_t \equiv \log \Pi_t$, and $\pi_t^* \equiv \log \Pi_t^*$, but now we take a first-order Taylor expansion of Eq. (1) with respect to the point $(\Pi_t = \Pi_t^*; \nu_t = \bar{\nu}_t)$. In doing so we assume that ν_t is stable around the possibly time-varying steady state $\bar{\nu}_t$, which captures persistent changes to the rule over time.[2]

Following similar steps to those taken before

$$\begin{aligned}
R_t =& \phi\left(\frac{\Pi_t}{\Pi_t^*}; \nu_t\right)\Bigg|_{\frac{\Pi_t}{\Pi_t^*}=1;\nu_t=\bar{\nu}_t} + \frac{\partial \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)}\Bigg|_{\frac{\Pi_t}{\Pi_t^*}=1;\nu_t=\bar{\nu}_t}\left(\frac{\Pi_t}{\Pi_t^*} - 1\right) \\
&+ \frac{\partial \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial \nu_t}\Bigg|_{\frac{\Pi_t}{\Pi_t^*}=1;\nu_t=\bar{\nu}_t}(\nu_t - \bar{\nu}_t) + \mathcal{O}(2) \\
e^{r_t} =& \phi_{0,t} + \frac{\partial \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)}\Bigg|_{1;\bar{\nu}_t}\left(e^{\pi_t - \pi^*} - 1\right) + \frac{\partial \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial \nu_t}\Bigg|_{1;\bar{\nu}_t}(\nu_t - \bar{\nu}_t) + \mathcal{O}(2) \\
1 + r_t =& \phi_{0,t} + \frac{\partial \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)}\Bigg|_{1;\bar{\nu}_t}(\pi_t - \pi_t^*) + \frac{\partial \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial \nu_t}\Bigg|_{1;\bar{\nu}_t}(\nu_t - \bar{\nu}_t) + \mathcal{O}(2),
\end{aligned}$$

and defining the following time-varying quantities

$$r_t^* \equiv \phi_{0,t} - 1 \equiv \phi\left(\frac{\Pi_t}{\Pi_t^*}; \nu_t\right)\Bigg|_{\frac{\Pi_t}{\Pi_t^*}=1;\nu_t=\bar{\nu}_t} - 1,$$

[2] This expansion necessarily assumes that the exogenous variables ν_t and Π_t remain for a long time within the neighbourhood of what can be thought of as their steady-state values.

$$\phi_t \equiv \left.\frac{\partial\phi\left(\Pi_t/\Pi_t^*;\nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)}\right|_{1;\bar{\nu}_t}, \qquad \sigma_t^{mp} = \left.\frac{\partial\phi\left(\Pi_t/\Pi_t^*;\nu_t\right)}{\partial\nu_t}\right|_{1;\bar{\nu}_t},$$

we obtain a Taylor rule, with time-varying coefficients, i.e.[3]

$$r_t = r_t^* + \phi_t(\pi_t - \pi_t^*) + \sigma_t^{mp}\nu_t^{mp} + \mathcal{O}(2), \tag{3}$$

where r_t^* can be thought of as the nominal equilibrium rate, and ϕ_t is the elasticity of the policy rule with respect to deviations of inflation from the target.[4]

Let us make three related remarks. First, the time variations in the coefficients of the Taylor rule depends on the potential time variation due to ν_t, which potentially affects all the parameters, and that has been modelled as the sum of a persistent and a non-persistent component, i.e. $\nu_t = \bar{\nu}_t + \nu_t^{mp}$. Second, for the log-linear expansion to be valid it has to be that the rule is relatively stable and the changes are not 'too large' otherwise the linear expansion around the time-varying steady state could be not valid and second order terms could be as large as or larger than first-order terms.[5] Third, and as a consequence, the time-varying nature of the coefficient needs a specification of the law of motion to assess the expansion since all terms with time-varying coefficients are potentially containing sums of first and second order terms.

We need now to focus on the role of $\bar{\nu}_t$ which we have been thinking of as the persistent part of the parameter characterising variations in policy or its implementation. Given the relative stability of the US monetary policy, we can think of it as a stochastic parameter

[3] The time-variation that has been captured through ν_t does not need to occur simultaneously in all the parameters of the Taylor rule and will depend on the nature of ϕ and the dimension of ν_t.

[4] The equation can be rewritten as

$$r_t = \bar{r}_t + \phi_t\pi_t + \mathcal{O}(2),$$

where $\bar{r} \equiv r_t^* - \phi_t\pi_t^* + \sigma_t^{mp}\nu_t^{mp}$ is a measure of the total exogenous shift in the central bank's reaction function.

[5] The inclusion of the random disturbances gives a log-linear (first-order Taylor series) approximation to that solution, accurate up to a residual of order $\mathcal{O}(||\xi||^2)$ where $||\xi||$ indexes the bounds on a given disturbance process ξ.

evolving as a bounded random walk (see, for example, Nicolau, 2002).[6] This assumption is in line with the intuition of a drift in the Taylor parameter, proposed by Bauer and Swanson (2023a,b).

This observation allows us to consider a further expansion in ν_t around what we can think of the central point of the bounded random walk. If the area in which the process behaves like a random walk is not 'too large', then a Taylor expansion can provide an approximation to the policy rule. Let us focus on the inflation parameter of the Taylor rule and consider an expansion at the centre of the bounded area of the bounded random walk process, $\bar{\nu}$, i.e.

$$\phi_t = \phi_t\Big|_{\bar{\nu}_t=\bar{\nu}} + \frac{\partial \phi_t}{\partial \bar{\nu}_t}\Big|_{\bar{\nu}_t=\bar{\nu}} (\bar{\nu}_t - \bar{\nu}) + \mathcal{O}(2), \tag{5}$$

$$= \frac{\partial \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)}\Big|_{1;\bar{\nu}} + \frac{\partial^2 \phi\left(\Pi_t/\Pi_t^*; \nu_t\right)}{\partial\left(\Pi_t/\Pi_t^*\right)\partial \bar{\nu}_t}\Big|_{1;\bar{\nu}} (\bar{\nu}_t - \bar{\nu}) + \mathcal{O}(2) \tag{6}$$

$$\equiv \phi + \widehat{\phi}_t + \mathcal{O}(2) \tag{7}$$

The expansion shows that we are now considering second-order terms, going beyond a first-order expansion. A similar expansion can be considered for the other time-varying parameters of the Taylor rule in Eq. (3). However, for the sake of the exposition, let us focus on the ϕ parameter only.

Following the described steps, we obtain:

$$r_t = r^* + (\phi + \widehat{\phi}_t)(\pi_t - \pi^*) + \sigma_{mp}\nu_t^{mp} + \mathcal{O}(2), \tag{8}$$

where $\mathcal{O}(2)$ now represents the remainder term which still contains second order terms, i.e.

[6] A possible way to write a bounded random walk process is the following, proposed by Nicolau (2002)

$$\bar{\nu}_t = \bar{\nu}_{t-1} + e^{\kappa}\left(e^{-\delta(\bar{\nu}_{t-1}-\bar{\nu})} - e^{\beta(\bar{\nu}_{t-1}-\bar{\nu})}\right) + \sigma_\varepsilon \varepsilon_t, \tag{4}$$

for $\delta \geq 0$, $\beta \geq 0$, $\kappa > 0$ where $\{\varepsilon_t\}$ is a sequence of independent and identically distributed (i.i.d.) random variables with $E[\varepsilon_t] = 0$ and $\text{Var}[\varepsilon_t] = 1$. The parameters κ, δ, and β are selected so that when ν_t is close to $\bar{\nu}$ it behaves as a random walk, while when it moves significantly away from it, it will be reversion effects that pull it toward $\bar{\nu}$ again. A bounded random walk can be indistinguishable from a random walk, although stochastically bounded by an upper and lower finite limit.

$\sim \nu_t^{mp,2}$, $(\pi_t - \pi_t^*)^2$, and $(\pi_t - \pi_t^*)\nu_t^{mp}$, albeit the expansion now features some terms beyond the first order, i.e. $\widehat{\phi}_t(\pi_t - \pi)$.

Let us recap. Starting from a general nonlinear policy rule we derived a linearised Taylor with an inflation coefficient varying through time and with a shift behaving like a random walk, but effectively covering a bounded space (and hence being a stationary and ergodic process). It is important to stress that in doing so we have to consider second order terms beyond the standard first-order log-linearisation of the Taylor expansion. Other second-order terms, of the same magnitude, may have been dropped in the expansion. From the point of view of the agents in the model, the random walk nature of the process driving the time-change in the rule parameters implies the need to forecast using the parameters from the last period.

A.2 A simple term structure model

Let us now consider, following Ellingsen and Soderstrom (2001), a simple affine term structure model of the type discussed in Svensson (1997, 1999), and based on the expectation hypothesis that does not model term premium.[7]

The economy is described by a set of linear equations

$$\pi_t = \pi_{t-1} + \iota y_{t-1} + \sigma_\pi u_t^\pi, \tag{9}$$

$$y_t = \hat{\beta} y_{t-1} - \delta(r_{t-1} - E_{t-1}[\pi_t]) + \sigma_y u_t^y, \tag{10}$$

$$r_t \equiv i_t^{(0)} = (\phi + \phi_t)\pi_t + (\omega + \omega_t)y_t + \sigma_{mp}\nu_t^{mp} \ , \tag{11}$$

$$i_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[r_{t+i}] + \xi_t^{(n)}, \tag{12}$$

$$\nu_t^{mp} = \zeta \nu_{t-1}^{mp} + u_t^{mp}, \tag{13}$$

where all the variables are considered in deviation from their steady state. The model features an accelerationist Phillips curve in which the change in the inflation rate is positively related

[7] We refer to Ellingsen and Soderstrom (2001) for a discussion of some of the standard results reported in this section.

to the previous period's output gap (Eq. 9), with $\iota > 0$, and u_t^π representing an i.i.d. supply shock with mean zero. The output gap is mean reverting and negatively related to the ex ante real short interest rate (Eq. 10). By substituting expectations of Eq. (9) in Eq. (10), one gets

$$y_t = \beta y_{t-1} - \delta(r_{t-1} - \pi_{t-1}) + \sigma_y u_t^y, \tag{14}$$

for $\beta \equiv \hat{\beta} + \delta\iota$. The short term interest rate is set according to a Taylor rule that responds to inflation and output gap (Eq. 16), and possibly with time-varying parameters as discussed in the previous section. The yield curve is specified as following the expectation hypothesis, and the n-periods ahead interest rate given by the expected path of the short term interest rate plus an exogenous term premium, $\xi_t^{(n)}$ (Eq. 12). Finally, the monetary policy disturbance, ν_t^{mp}, follows an autoregressive process of order one (Eq. 13).

A.2.1 The central bank's problem

As discussed in Ellingsen and Soderstrom (2001), the policy equation can be obtained from the problem of a central bank trying to minimise a loss function

$$\mathcal{L} = E_t \sum_{i=0}^{\infty} \vartheta^i \frac{1}{2} \left(\pi_{t+s}^2 + \lambda_t y_{t+s}^2 \right), \tag{15}$$

where the parameter λ_t is the weight of output stabilisation relative to inflation stabilisation, which in our setting may change over time. The solution of this programme for the central bank delivers a linear policy rule

$$r_t = \phi \pi_t + \omega y_t + \sigma_{mp} \nu_t^{mp} \tag{16}$$

with coefficients

$$\phi = 1 + \frac{\iota \vartheta k}{\delta(\lambda_t + \iota^2 \vartheta k)}, \qquad \omega = \frac{\beta}{\delta} + \iota(\phi - 1), \tag{17}$$

for $k > 1$ a constant.

How do the parameters change for a change to λ_t? Let us consider a first order expansion at a given value of λ, and define

$$\phi_t \equiv \frac{\partial \phi}{\partial \lambda_t}(\lambda_t - \lambda) = -\frac{\iota \vartheta k}{\delta(\lambda_t + \iota^2 \vartheta k)^2}(\lambda_t - \lambda) \tag{18}$$

$$\omega_t \equiv \frac{\partial \omega}{\partial \lambda_t}(\lambda_t - \lambda) = \iota \frac{\partial \phi}{\partial \lambda_t}(\lambda_t - \lambda) = -\iota^2 \frac{\vartheta k}{\delta(\lambda_t + \iota^2 \vartheta k)^2}(\lambda_t - \lambda) \tag{19}$$

If we assume that λ_t evolves in a neighbourhood of λ as a bounded random walk, then ϕ_t and ω_t inherit the bounded random walk dynamics

$$\phi_t = \phi_{t-1} + \sigma_\phi u_t^\phi, \tag{20}$$

$$\omega_t = \omega_{t-1} + \iota \sigma_\phi u_t^\phi. \tag{21}$$

In the following discussion of this model, we will focus on the solution of the model for $\phi_t = \omega_t = 0$ and consider how a shift to the policy rule parameters, as well as different shocks affect the yield curve.

A.2.2 The yield curve in the economy

To find a solution for the yield curve, we can start by taking expectations of the policy rate, in Eq. (12), i periods ahead:

$$E_t[r_{t+i}] = \phi E_t[\pi_{t+i}] + \omega E_t[y_{t+i}] + \sigma_{mp} E_t[\nu_{t+i}^{mp}] \tag{22}$$

By employing Eq. (9), (16) and (14), we obtain:

$$
\begin{aligned}
E_t[y_{t+i}] &= \beta E_t[y_{t+i-1}] - \delta(E_t[r_{t+i-1}] - E_t[\pi_{t+i-1}]) \\
&= -\delta(\phi-1)E_t[\pi_{t+i-1}] + (\beta - \delta\omega)E_t[y_{t+i-1}] - \delta E_t[\sigma_{mp}\nu^{mp}_{t+i-1}] \\
&= -\delta(\phi-1)E_t[\pi_{t+i-1}] - \delta\iota(\phi-1)E_t[y_{t+i-1}] - \delta E_t[\sigma_{mp}\nu^{mp}_{t+i-1}] \\
&= -\delta(\phi-1)E_t[\pi_{t+i}] - \delta E_t[\sigma_{mp}\nu^{mp}_{t+i-1}] \qquad (23)
\end{aligned}
$$

Likewise, the expected future inflation rate at period $t+i$:

$$
\begin{aligned}
E_t[\pi_{t+i}] &= E_t[\pi_{t+i-1}] + \iota E_t[y_{t+i-1}] = (1-\iota\delta(\phi-1))E_t[\pi_{t+i-1}] - \delta\iota E_t[\sigma_{mp}\nu^{mp}_{t+i-2}] \\
&= (1-\iota\delta(\phi-1))^2 E_t[\pi_{t+i-2}] - (1-\iota\delta(\phi-1))\delta\iota E_t[\sigma_{mp}\nu^{mp}_{t+i-3}] - \delta\iota E_t[\sigma_{mp}\nu^{mp}_{t+i-2}] \\
&= (1-\iota\delta(\phi-1))^{i-1}[\pi_t + \iota y_t] - \delta\iota\big[\sum_{j=0}^{i-2}(1-\iota\delta(\phi-1))^j \zeta^{i-2-j}\big]\sigma_{mp}\nu^{mp}_t \\
&= (1-\iota\delta(\phi-1))^{i-1}[\pi_t + \iota y_t] - \delta\iota\frac{\zeta^{i-1} - [1-\iota\delta(\phi-1)]^{i-1}}{\zeta - [1-\iota\delta(\phi-1)]}\sigma_{mp}\nu^{mp}_t, \qquad (24)
\end{aligned}
$$

where, in the second line, we observe that

$$
E_t[\nu^{mp}_{t+i}] = \zeta E_t[\nu^{mp}_{t+i-1}] = ... = \zeta^i \nu^{mp}_t. \qquad (25)
$$

The condition

$$
|1 - \iota\delta(\phi-1)| < 1,
$$

is needed to get a finite sum in the above summations.

We can now substitute Eq. (24) into Eq. (23) to obtain

$$
\begin{aligned}
E_t[y_{t+i}] = -\delta(\phi-1)\Bigg[&(1-\iota\delta(\phi-1))^{i-1}[\pi_t + \iota y_t] \\
&- \delta\iota\frac{\zeta^{i-1} - [1-\iota\delta(\phi-1)]^{i-1}}{\zeta - [1-\iota\delta(\phi-1)]}\sigma_{mp}\nu^{mp}_t\Bigg] - \delta\zeta^{i-1}\sigma_{mp}\nu^{mp}_t. \qquad (26)
\end{aligned}
$$

By substituting Eq. (24) and Eq. (26) in Eq. (22), we obtain the following expression

$$E_t[r_{t+i}] = [\phi - \omega\delta(\phi-1)]\Bigg[(1-\iota\delta(\phi-1))^{i-1}[\pi_t + \iota y_t] \\ - \delta\iota\frac{\zeta^{i-1} - [1-\iota\delta(\phi-1)]^{i-1}}{\zeta - [1-\iota\delta(\phi-1)]}\sigma_{mp}\nu_t^{mp}\Bigg] + (\zeta^i - \omega\delta\zeta^{i-1})\sigma_{mp}\nu_t^{mp}. \quad (27)$$

We can now observe that:

$$\sum_{i=1}^{n-1} E_t[r_{t+i}] = [\phi - \omega\delta(\phi-1)]\left[\sum_{i=1}^{n-1}(1-\iota\delta(\phi-1))^{i-1}\right](\pi_t + \iota y_t) \\ - \frac{[\phi-\omega\delta(\phi-1)]\delta\iota}{\zeta - [1-\iota\delta(\phi-1)]}\left[\sum_{i=1}^{n-1}(\zeta^{i-1} - [1-\iota\delta(\phi-1)]^{i-1})\right]\sigma_{mp}\nu_t^{mp} \\ + \left[\sum_{i=1}^{n-1}(\zeta^i - \omega\delta\zeta^{i-1})\right]\sigma_{mp}\nu_t^{mp} \quad (28)$$

and

$$\sum_{i=1}^{n-1}(1-\iota\delta(\phi-1))^{i-1} = \sum_{i=0}^{n-2}(1-\iota\delta(\phi-1))^{i} = \frac{1-(1-\iota\delta(\phi-1))^{n-2+1}}{\iota\delta(\phi-1)} = \frac{1-(1-\iota\delta(\phi-1))^{n-1}}{\iota\delta(\phi-1)}$$

$$\sum_{i=1}^{n-1}(\zeta^{i-1} - [1-\iota\delta(\phi-1)]^{i-1}) = \sum_{i=0}^{n-2}(\zeta^{i} - [1-\iota\delta(\phi-1)]^{i}) = \frac{1-\zeta^{n-1}}{1-\zeta} - \frac{1-(1-\iota\delta(\phi-1))^{n-1}}{\iota\delta(\phi-1)}$$

$$\sum_{i=1}^{n-1}(\zeta^i - \omega\delta\zeta^{i-1}) = \zeta\frac{1-\zeta^{n-1}}{1-\zeta} - \omega\delta\frac{1-\zeta^{n-1}}{1-\zeta} = (\zeta - \omega\delta)\frac{1-\zeta^{n-1}}{1-\zeta}$$

Thus, we obtain:

$$\sum_{i=1}^{n-1} E_t[r_{t+i}] = \frac{[\phi-\omega\delta(\phi-1)]\left[1-(1-\iota\delta(\phi-1))^{n-1}\right]}{\iota\delta(\phi-1)}(\pi_t + \iota y_t) + \\ - \frac{[\phi-\omega\delta(\phi-1)]\delta\iota}{\zeta-[1-\iota\delta(\phi-1)]}\left[\frac{1-\zeta^{n-1}}{1-\zeta} - \frac{1-[1-\iota\delta(\phi-1)]^{n-1}}{\iota\delta(\phi-1)}\right]\sigma_{mp}\nu_t^{mp} \\ + \frac{(\zeta-\omega\delta)(1-\zeta^{n-1})}{1-\zeta}\sigma_{mp}\nu_t^{mp}. \quad (29)$$

Hence, from the term structure in Eq. (12), we obtain an analytical solution for bonds at different maturities, i.e.

$$
\begin{aligned}
i_t^{(n)} = \frac{1}{n}\Bigg\{ & \phi\pi_t + \omega y_t + \sigma_{mp}\nu_t^{mp} + \frac{[\phi-\omega\delta(\phi-1)]\left[1-(1-\iota\delta(\phi-1))^{n-1}\right]}{\iota\delta(\phi-1)}(\pi_t+\iota y_t) \\
& - \frac{(\phi-\omega\delta(\phi-1))\,\delta\iota}{\zeta-(1-\iota\delta(\phi-1))}\left[\frac{1-\zeta^{n-1}}{1-\zeta} - \frac{1-(1-\iota\delta(\phi-1))^{n-1}}{\iota\delta(\phi-1)}\right]\sigma_{mp}\nu_t^{mp} \\
& + \frac{(\zeta-\omega\delta)(1-\zeta^{n-1})}{1-\zeta}\,\sigma_{mp}\nu_t^{mp}\Bigg\} + \xi_t^{(n)}, \quad (30)
\end{aligned}
$$

The yield curve can be written in the standard form of an affine model as

$$
i_t^{(n)} = a_n + b_n^{\pi}\pi_t + b_n^{y} y_t + c_n\nu_t^{mp} + \xi_t^{(n)}, \quad (31)
$$

with $a_n \equiv 0$ and

$$
b_n^{\pi} \equiv \frac{1}{n}\left(\phi + \frac{(\phi-\omega\delta(\phi-1))\left(1-(1-\iota\delta(\phi-1))^{n-1}\right)}{\iota\delta(\phi-1)}\right) \quad (32)
$$

$$
b_n^{y} \equiv \frac{1}{n}\left(\omega + \iota\frac{(\phi-\omega\delta(\phi-1))\left(1-(1-\iota\delta(\phi-1))^{n-1}\right)}{\iota\delta(\phi-1)}\right) \quad (33)
$$

$$
\begin{aligned}
c_n \equiv \frac{\sigma_{mp}}{n}\Bigg\{ & 1 - \frac{(\phi-\omega\delta(\phi-1))\,\delta\iota}{\zeta-(1-\iota\delta(\phi-1))}\left(\frac{1-\zeta^{n-1}}{1-\zeta} - \frac{1-(1-\iota\delta(\phi-1))^{n-1}}{\iota\delta(\phi-1)}\right) \\
& + \frac{(\zeta-\omega\delta)(1-\zeta^{n-1})}{1-\zeta}\Bigg\}
\end{aligned} \quad (34)
$$

A.2.3 The impact of a policy parameter shift

The yield change triggered by the monetary policy announcement, conditional on a shock to the policy parameters, u_t^{ϕ}, can be written as

$$
\Delta_{[\bar{t}-\underline{t}]} i_t^{(n)} = \frac{\partial i_t^{(n)}}{\partial\phi_t}\frac{\phi_t}{\partial u_t^{\phi}} = \sigma_{\phi}\frac{\partial i_t^{(n)}}{\partial\phi_t}u_t^{\phi}, \quad (35)
$$

where $\Delta_{[\bar{t}-\underline{t}]}$ indicates the difference between the yields before and after the announcement. To compute the changes of the yields triggered by a change in the time-varying part of the policy parameter, we can consider the model with constant ϕ and take a derivative in that parameter.

To compute the derivative of the rates with respect to a change in the parameter of the Taylor rule ϕ, i.e. $\frac{\partial i_t^{(n)}}{\partial \phi}$, let us consider Eq. (30) and set the monetary policy shock to zero. At $n = 1$, the policy rate responds as

$$\frac{\partial i_t^{(1)}}{\partial \phi} = \pi_t + \iota y_t,$$

and hence the impact is positive and equal to one. For $n \geq 2$, we need a bit more work, let us focus on the coefficient in front of $\pi + \iota y_t$, which we can rewrite as

$$\frac{[\phi - \omega\delta(\phi - 1)]\left[1 - (1 - \iota\delta(\phi - 1))^{n-1}\right]}{\iota\delta(\phi - 1)} = \frac{T_1(\phi)T_2(\phi)}{T_3(\phi)}. \tag{36}$$

where we define

$$T_1(\phi) \equiv [\phi - \omega\delta(\phi - 1)], \quad T_2(\phi) \equiv \left[1 - (1 - \iota\delta(\phi - 1))^{n-1}\right], \quad T_3(\phi) \equiv \iota\delta(\phi - 1). \tag{37}$$

Using the standard derivation rules

$$\begin{aligned}\frac{d}{d\phi}\left(\frac{T_1(\phi)T_2(\phi)}{T_3(\phi)}\right) &= \frac{T_3(\phi)\frac{d}{d\phi}[T_1(\phi)T_2(\phi)] - T_1(\phi)T_2(\phi)\frac{dT_3(\phi)}{d\phi}}{T_3(\phi)^2} \\ &= \frac{\iota\delta(\phi - 1)\left[(1 - \iota\delta(\phi - 1) - \omega\delta)T_2(\phi) + T_1(\phi)(n-1)\left(1 - \iota\delta(\phi - 1)\right)^{n-2}\iota\delta\right]}{\left(\iota\delta(\phi - 1)\right)^2} \\ &\qquad - \frac{T_1(\phi)T_2(\phi)\iota\delta}{\left(\iota\delta(\phi - 1)\right)^2},\end{aligned}$$

where we used

$$\frac{dT_1(\phi)}{d\phi} = 1 - \iota\delta(\phi - 1) - \omega\delta, \quad \frac{dT_2(\phi)}{d\phi} = (n-1)\left(1 - \iota\delta(\phi - 1)\right)^{n-2}\iota\delta, \quad \frac{dT_3(\phi)}{d\phi} = \iota\delta, \tag{38}$$

and the fact that $\frac{\partial \omega}{\partial \phi} = \iota$. As observed above, the derivative of the term $\phi \pi_t + \omega y_t$ in ϕ is equal to $\pi_t + \iota y_t$. The derivative of the rates at different maturities is therefore

$$\frac{\partial i_t^{(n)}}{\partial \phi} = \frac{1}{n} \left\{ 1 + \frac{1}{\iota \delta(\phi - 1)} \Big[(1 - \iota\delta(\phi-1) - \omega\delta) T_2(\phi) \right. \\ \left. + T_1(\phi)(n-1)\left(1 - \iota\delta(\phi-1)\right)^{n-2} \iota\delta - \frac{T_1(\phi)T_2(\phi)}{\phi - 1} \Big] \right\} (\pi_t + \iota y_t). \quad (39)$$

For n large, since $(1 - \iota\delta(\phi - 1)) < 1$, we get $T_2 \sim 1$,

$$T_1(\phi)(n-1)\left(1 - \iota\delta(\phi-1)\right)^{n-2} \sim 0,$$

and

$$T_1(\phi)T_2(\phi) = \left[\phi - \omega\delta(\phi-1)\right]\left[1 - \left(1 - \iota\delta(\phi-1)\right)^{n-1}\right] \sim \left[\phi - \omega\delta(\phi-1)\right],$$

hence

$$\begin{aligned} \frac{\partial i_t^{(n)}}{\partial \phi} &\sim \frac{1}{n} \left\{ 1 + \frac{1}{\iota\delta(\phi-1)} \left[(1 - \iota\delta(\phi-1) - \omega\delta) - \frac{\phi - \omega\delta(\phi-1)}{\phi - 1} \right] \right\} (\pi_t + \iota y_t) \\ &= \frac{1}{n} \left\{ 1 + \frac{1}{\iota\delta(\phi-1)^2} \left[(\phi - 1) - \iota\delta(\phi-1)^2 - \phi \right] \right\} (\pi_t + \iota y_t) \\ &= -\frac{1}{n} \left\{ \frac{1}{\iota\delta(\phi-1)^2} \right\} (\pi_t + \iota y_t) \quad (40) \end{aligned}$$

Thus, conditional on positive values of inflation and output at time t, a shift in ϕ causes short maturities to rise and longer maturities to decrease. This results and the following proposition are similar to what derived by Ellingsen and Soderstrom (2001).

Lemma 1 (Shock to the parameters of the policy rule). *If a policy decision of the central bank reveals a change in the preferences of the central bank, interest rates on bonds of sufficiently long maturity move in the opposite direction to the unexpected change in the policy rate, i.e. an unexpectedly high central-bank rate tilts the yield curve clockwise, while an unexpectedly low rate tilts it counterclockwise.*

A.2.4 The impact of information

Following Ellingsen and Soderstrom (2001), let us consider a situation in which agents are not perfectly informed and the effect of information conveyed by a policy action, i.e. agents learning about a demand and/or a supply shock via the central bank's decision. The yield change triggered by information about the shocks can be written as

$$\Delta_{[\bar{t}-\underline{t}]} i_t^{(n)} = \frac{\partial i_t^{(n)}}{\partial u_t^{\pi}} u_t^{\pi} + \frac{\partial i_t^{(n)}}{\partial u_t^{y}} u_t^{y}. \tag{41}$$

Differentiating $i_t^{(n)}$ in Eq. (30) with respect to u_t^{π}, one obtains

$$\frac{\partial i_t^{(n)}}{\partial u_t^{\pi}} = \frac{\sigma_\pi}{n} \left\{ \phi + \frac{[\phi - \omega\delta(\phi - 1)] \left[1 - (1 - \iota\delta(\phi - 1))^{n-1}\right]}{\iota\delta(\phi - 1)} \right\}, \tag{42}$$

while taking a derivative in the demand shock, u_t^{y}, one finds

$$\frac{\partial i_t^{(n)}}{\partial u_t^{y}} = \frac{\sigma_y}{n} \left\{ \omega + \iota \frac{[\phi - \omega\delta(\phi - 1)] \left[1 - (1 - \iota\delta(\phi - 1))^{n-1}\right]}{\iota\delta(\phi - 1)} \right\}. \tag{43}$$

Since $\phi > 1$, and $(1 - \iota\delta(\phi - 1)) < 1$, we can conclude that an information shock would lift all the maturities with the magnitude of the effects decreasing with n^{-1}.

Lemma 2 (Information effects). *If a policy decision of the central bank transmits information about either a supply or a demand shock, market interest rates will comove with the policy rate change at all maturities, with the magnitude of the effects decreasing over the maturities at rate n^{-1}.*

A.2.5 The impact of monetary policy shock

The effect of a monetary policy shock on the yield curve can be written as

$$\Delta_{[\bar{t}-\underline{t}]} i_t^{(n)} = \frac{\partial i_t^{(n)}}{\partial \nu_t^{mp}} u_t^{mp}, \tag{44}$$

where the impact of a monetary policy disturbance is given by

$$\frac{\partial i_t^{(n)}}{\partial \nu_t^{mp}} = \frac{\sigma_{mp}}{n}\left\{1 - \frac{[\phi - \omega\delta(\phi-1)]\delta\iota}{\zeta - (1-\iota\,\delta(\phi-1))}\left[\frac{1-\zeta^{n-1}}{1-\zeta}\right.\right.$$
$$\left.\left. - \frac{1-(1-\iota\,\delta(\phi-1))^{n-1}}{\iota\delta(\phi-1)}\right] + \frac{(\zeta-\omega\delta)(1-\zeta^{n-1})}{1-\zeta}\right\} \quad (45)$$

On the policy rate, the effect of the shock is positive and equal to σ_{mp}, i.e.

$$\frac{\partial i_t^{(1)}}{\partial \nu_t^{mp}} = \sigma_{mp}. \quad (46)$$

For n large, since $1 - \iota\,\delta(\phi-1) < 1$ and $|\zeta| < 1$, the leading terms are

$$\frac{\partial i_t^{(n)}}{\partial \nu_t^{mp}} \sim \frac{\sigma_{mp}}{n}\left\{1 - \frac{[\phi - \omega\delta(\phi-1)]\delta\iota}{\zeta - a}\left[\frac{1}{1-\zeta} - \frac{1}{\iota\delta(\phi-1)}\right] + \frac{\zeta-\omega\delta}{1-\zeta}\right\}$$
$$= \frac{\sigma_{mp}}{n}\left[1 - \frac{1}{1-\zeta}\left(\frac{\phi}{\phi-1} - \zeta\right)\right] = -\frac{\sigma_{mp}}{n}\frac{1}{(\phi-1)(1-\zeta)}, \quad (47)$$

which is always negative since $\phi > 1$, and goes to zero for n large.

To summarise: (i) the response to a monetary policy shock in this model goes to zero with n^{-1}, (ii) the impact of a positive monetary policy shock is positive at short maturities to become negative at long maturities.

Lemma 3 (Monetary policy shock). *Following a monetary policy shock the interest rates on bonds of sufficiently long maturity will move in the opposite direction to the monetary policy shock and the movement of the short maturities. The magnitude of the effects declines with the maturity of the bond, at rate n^{-1}.*

A.2.6 A discussion on the magnitude of the effects at the end of the curve

As we have seen the effects for both a monetary policy shock and a change to the rule, conditionally on a demand or supply shock, are declining over the maturities and have opposite in sign effect on shorter maturities and long maturities. However, the magnitude can be very different. The intuition for this remark is that a monetary policy shock is expected to dissipate at business cycle maturities leaving the long end of the curve unaffected. Conversely, a shift to the parameters of the policy rule can impress a stronger rotation on the long-end of the curve.

The results reported above support this intuition, in fact, the impact of a shift to the rule parameters onto yields with long maturities (i.e. for large n), conditional on a demand (or supply shock) is

$$\Delta_{[\bar{t}-\underline{t}]} i_t^{(n)} \sim -\frac{\sigma_\phi}{n} \left\{ \frac{1}{\iota\delta(\phi-1)^2} \right\} u_t^\phi (\sigma_\pi u_t^\pi + \iota\sigma_y, u_t^y)$$

while for a monetary policy shock, we obtained

$$\Delta_{[\bar{t}-\underline{t}]} i_t^{(n)} \sim -\frac{\sigma_{mp}}{n} \frac{1}{(\phi-1)(1-\zeta)} u_t^{mp}.$$

Hence conditionally on unit shocks (and unit variances), we need to compare

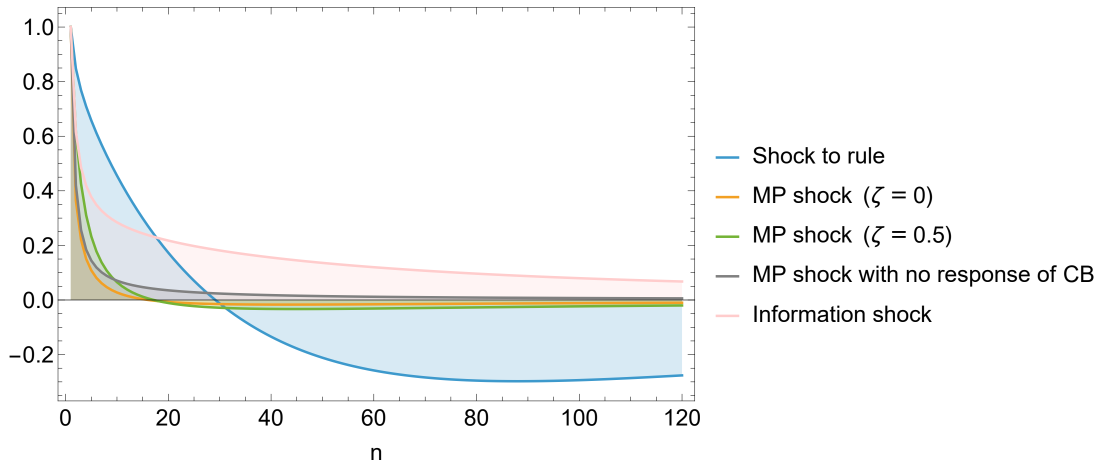
$$\frac{1}{\iota\delta(\phi-1)^2} \qquad \text{vs.} \qquad \frac{1}{(\phi-1)(1-\zeta)},$$

which for a standard range of the parameters gives ι, δ and $(\phi-1)$ between zero and one. Therefore while at longer maturities a monetary policy shock has effects of roughly an order of magnitude 10^2 smaller than a shift to the rule parameters.

For the standard range of parameters, ι, δ and $(\phi-1)$ lie between zero and one. Therefore, at longer maturities, the effect of a monetary policy shock is approximately two orders of magnitude (10^2) smaller than that of a shift in the rule parameters. Figure (A.1), plots the impact of the different shocks, for a set of parameters similar to those used by Smith and Taylor (2009). It also reports a monetary policy shock in the case in which the central bank does not respond to the consequences of it own shock.

Lemma 4 (**Magnitude of the effects at the long end of yield curve).** *At longer maturities, the impact of a monetary policy shock on yields is significantly smaller than the*

Figure A.1: THE REACTION OF THE YIELD CURVE TO SHOCKS (MODEL I)



Notes: The figure compares the impact on the term structure of interest rates resulting from a shift in the policy rule (blue), a monetary policy shock (orange when $\zeta = 0$ and green when $\zeta = 0.5$), and an information shock (light red). The calibration follows Smith and Taylor (2009). In grey, the figure reports the term structure's reaction under the assumption that the central bank does not respond to the macroeconomic consequences of its own monetary policy shock, which follows an AR(1) process with an autocorrelation coefficient of 0.5, which follows an AR(1) process with an autocorrelation coefficient of 0.5. $n = 120$ are 120 quarters (i.e., 30 years).

impact of a shift in policy rule parameters. Specifically, for standard parameter values, the effect of a monetary policy shock declines at a rate of approximately 10^2 *times smaller than that of a change in policy parameters.*

A.3 Imperfect information and the yield curve

The model discussed in the previous section, following Ellingsen and Soderstrom (2001), captures only the expectations component of interest rates without accounting for term premium dynamics. Moreover, it does not explicitly model the information flow between the central bank and market participants. In this section, we introduce a stylised imperfect information framework that integrates the policy rule from the previous section into an affine term structure model with a term premium, as in Smith and Taylor (2009).[8]

All the variables are considered in deviation from their steady state, and their dynamics

[8] As compared to the previous model, for sake of simplicity was consider a policy rule that only responds to inflation. Results in Smith and Taylor (2009) show that the results on the response of the yield curve to shocks extends to the case of a more general policy rule reacting to output gap, and hence this applies also to our discussion.

is described by the following set of equations

$$r_t = (\phi + \widehat{\phi}_t)\pi_t + \sigma_{mp}\nu_t^{mp}, \tag{48}$$

$$\nu_t^{mp} = \zeta\nu_{t-1}^{mp} + u_t^{mp} \tag{49}$$

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \sigma_\phi u_t^\phi \tag{50}$$

$$i_t^{(n)} = -\frac{1}{n}\log P_t^{(n)}, \tag{51}$$

$$P_t^{(n+1)} = \mathbb{E}_t\left[m_{t+1}P_{t+1}^{(n)}\right], \tag{52}$$

$$m_{t+1} = e^{-r_t - \frac{1}{2}\lambda_t^2 - \lambda_t u_{t+1}^\pi}, \tag{53}$$

$$\lambda_t = -\gamma - \psi\pi_t, \tag{54}$$

$$\pi_t = \alpha\pi_{t-1} - \delta(r_{t-1} - \pi_{t-1}) + \sigma_\pi u_t^\pi. \tag{55}$$

Eq. (48) represents the (linearised) monetary policy rule, where the short-term nominal interest rate, r_t, responds to inflation with a policy response coefficient $\phi + \phi_t > 1$. Eq. (49) models the transitory policy shock as an AR(1) process with coefficient $0 < \zeta < 1$, capturing the policy inertia observed in the data. Eq. (51) defines the yield to maturity of a zero-coupon bond with face value one, maturing in n periods, where $P_t^{(n)}$ denotes the bond price at time t. Eq. (52) states a no-arbitrage condition, requiring that the price of an $n+1$-period bond at time t equals the expected present discounted value of an n-period bond at time $t+1$, given the stochastic discount factor m_t. Eq. (53) specifies the stochastic discount factor, adopting its functional form from the affine term structure literature. Eq. (54) models the risk factor as a combination of a constant risk premium, γ, and a time-varying risk premium, ψ, linked to inflation fluctuations. Finally, Eq. (86) describes inflation dynamics as a function of the lagged real interest rate and past inflation.

The model features three independent and identically distributed normal white noise shocks: an inflation shock, a conventional monetary policy shock, and a policy rule shifter, denoted as $u_t^i \sim$ i.i.d. $\mathcal{N}(0, 1)$ for $i \in (\pi, mp, \phi)$.

Although we consider shifts in the Taylor rule parameter, we assume that both the central bank and private agents perceive it as fixed and known at any given time. Consequently, any changes to the Taylor rule parameters are fully unanticipated and regarded as permanent, aligning with the intuition proposed by Bauer and Swanson (2023a,b).

This model admits an affine structure for the yield curve:

$$i_t^{(n)} = a_n + b_n \pi_t + c_n \nu_t^{mp}, \tag{56}$$

$$P_t^{(n)} = e^{A_n + B_n \pi_t + C_n \nu_t^{mp}}. \tag{57}$$

with the following relationships:

$$a_n = -\frac{A_n}{n}, \quad b_n = -\frac{B_n}{n}, \quad c_n = -\frac{C_n}{n}. \tag{58}$$

A.3.1 Solution by the method of undetermined coefficients

We solve the model using the method of undetermined coefficients. For $n = 1$, where $i_t^{(1)} = r_t$, Eq. (48) yields

$$a_1 = 0, \quad A_1 = 0, \tag{59}$$

$$b_1 = \phi, \quad B_1 = -\phi, \tag{60}$$

$$c_1 = \sigma_{mp}, \quad C_1 = -\sigma_{mp}. \tag{61}$$

From Eq. (52) one obtains

$$
\begin{aligned}
P_t^{(n+1)} &= \mathbb{E}_t\left[m_{t+1}P_{t+1}^{(n)}\right] \\
&= \mathbb{E}_t\left[e^{-r_t-\frac{1}{2}\lambda_t^2-\lambda_t u_{t+1}^{\pi}}e^{A_n+B_n\pi_{t+1}+C_n\nu_{t+1}^{mp}}\right] \\
&= \mathbb{E}_t\left[e^{-\phi\pi_t-\sigma_{mp}\nu_t^{mp}-\frac{1}{2}\lambda_t^2-\lambda_t u_{t+1}^{\pi}+A_n+B_n\pi_{t+1}+C_n\nu_{t+1}^{mp}}\right] \\
&= \mathbb{E}_t\left[e^{-\phi\pi_t-\sigma_{mp}\nu_t^{mp}-\frac{1}{2}\lambda_t^2-\lambda_t u_{t+1}^{\pi}+A_n+B_n\left(\alpha\pi_t-\delta(\phi\pi_t+\sigma_{mp}\nu_t^{mp}-\pi_t)+\sigma_\pi u_{t+1}^{\pi}\right)+C_n(\zeta\nu_t^{mp}+u_{t+1}^{mp})}\right] \\
&= e^{-\phi\pi_t-\sigma_{mp}\nu_t^{mp}-\frac{1}{2}\lambda_t^2+A_n+B_n\left(\alpha\pi_t-\delta(\phi\pi_t+\sigma_{mp}\nu_t^{mp}-\pi_t)\right)+\zeta C_n\nu_t^{mp}}\mathbb{E}_t\left[e^{-\lambda_t u_{t+1}^{\pi}+B_n\sigma_\pi u_{t+1}^{\pi}+C_n u_{t+1}^{mp}}\right] \\
&= e^{-\phi\pi_t+(\zeta C_n-\sigma^{mp})\nu_t^{mp}-\frac{1}{2}\lambda_t^2+A_n+B_n\left(\alpha\pi_t-\delta(\phi\pi_t+\sigma_{mp}\nu_t^{mp}-\pi_t)\right)+\frac{1}{2}\left(\lambda_t^2+B_n^2\sigma_\pi^2+C_n^2\right)-\lambda_t B_n\sigma_\pi} \\
&= e^{-\phi\pi_t+(\zeta C_n-(1+B_n\delta))\sigma_{mp}\nu_t^{mp}+A_n+B_n(\alpha+\delta(1-\phi))\pi_t+\frac{1}{2}\left(B_n^2\sigma_\pi^2+C_n^2\right)+B_n\sigma_\pi(\gamma+\psi\pi_t)} \\
&= e^{A_n+\frac{1}{2}\left(B_n^2\sigma_\pi^2+C_n^2\right)+B_n\sigma_\pi\gamma+(-\phi+B_n(\alpha+\delta(1-\phi)+\sigma_\pi\psi))\pi_t+(\zeta C_n-(1-B_n\delta))\sigma_{mp}\nu_t^{mp}}
\end{aligned}
$$

where the equalities are derived by sequentially substituting the no-arbitrage condition, Eq. (53), Eq. (57), Eq. (48), and Eq. (86); then taking the expected value of the exponential of a normally distributed variable, and finally using Eq. (54) before simplifying the expression and factorising.

Matching coefficients with Eq. (57), one obtains the following set of recursive equations for the coefficients:

$$A_{n+1} = A_n + \frac{1}{2}\left(B_n^2\sigma_\pi^2 + C_n^2\right) + B_n\sigma_\pi\gamma \tag{62}$$

$$B_{n+1} = -\phi + B_n(\alpha + \delta(1-\phi) + \sigma_\pi\psi) \tag{63}$$

$$C_{n+1} = \zeta C_n - \sigma_{mp}(1 + B_n\delta) \tag{64}$$

By using the initial conditions in (59)–(61), we can write:

$$B_n = -\phi \sum_{i=0}^{n-1} (\alpha + \delta(1-\phi) + \sigma_\pi \psi)^i \tag{65}$$

$$b_n = \frac{\phi}{n} \sum_{i=0}^{n-1} (\alpha + \delta(1-\phi) + \sigma_\pi \psi)^i \tag{66}$$

Substituting in the expression for C_n, we obtain

$$C_n = -\sigma_{mp} \left[\zeta^{n-1} + \sum_{j=1}^{n-1} \zeta^{n-1-j} \left(1 - \delta\phi \sum_{i=0}^{j-1} (\alpha + \delta(1-\phi) + \sigma_\pi \psi)^i \right) \right] \tag{67}$$

$$c_n = \frac{\sigma_{mp}}{n} \left[\zeta^{n-1} + \sum_{j=1}^{n-1} \zeta^{n-1-j} \left(1 - \delta\phi \sum_{i=0}^{j-1} (\alpha + \delta(1-\phi) + \sigma_\pi \psi)^i \right) \right] \tag{68}$$

Substituting in the equation for A_n, we obtain

$$A_{n+1} = A_n + \sigma_\pi \gamma B_n + \frac{1}{2} \left(\sigma_\pi^2 B_n^2 + \sigma_{mp}^2 \sum_{j=1}^{n-1} \zeta^j (1 + \delta B_j)^2 \right), \tag{69}$$

which can be solved iterating the difference equation.

The sum in Eq. (65) is finite under the condition

$$|\alpha + \delta(1-\phi) + \sigma_\pi \psi| < 1.$$

Restricting the parameter space for $\phi > 0$, the condition implies that

$$\phi > 1 + \frac{\alpha - 1 + \sigma_\pi \psi}{\delta}. \tag{70}$$

Thus, we assume that the random walk, in every period t, is bounded below by the term in equation (70). We can now simplify the above expressions by defining

$$\kappa \equiv \alpha + \delta(1-\phi) + \sigma_\pi \psi,$$

and using the standard formula for geometric sums, for $|\kappa| < 1$, to obtain

$$\begin{aligned} B_n &= -\phi \frac{1-\kappa^n}{1-\kappa}, & (71)\\ C_n &= -\sigma_{mp}\left[\zeta^{n-1} + \frac{1-\zeta^{n-1}}{1-\zeta} - \frac{\delta\phi}{1-\kappa}\left(\frac{1-\zeta^{n-1}}{1-\zeta} - \kappa\frac{\kappa^{n-1}-\zeta^{n-1}}{\kappa-\zeta}\right)\right], & (72)\end{aligned}$$

and

$$\begin{aligned} b_n &= \frac{\phi}{n}\frac{1-\kappa^n}{1-\kappa}, & (73)\\ c_n &= \frac{\sigma_{mp}}{n}\left[\zeta^{n-1} + \frac{1-\zeta^{n-1}}{1-\zeta} - \frac{\delta\phi}{1-\kappa}\left(\frac{1-\zeta^{n-1}}{1-\zeta} - \kappa\frac{\kappa^{n-1}-\zeta^{n-1}}{\kappa-\zeta}\right)\right]. & (74)\end{aligned}$$

Let us summarise these results in the following proposition.

Lemma 5 (Yield curve). *The yield curve described by the model in Eq.s (48-86) is*

$$i_t^{(n)} = a_n + b_n\pi_t + c_n u_t^{mp} \tag{75}$$

with coefficients of the disturbances given by

$$b_n = \frac{\phi\sum_{i=1}^{n-1}\kappa^i}{n}, \quad c_n = \frac{\sigma_{mp}\left(1-\delta\phi\sum_{i=0}^{n-2}\kappa^i\right)}{n}, \tag{76}$$

for $\kappa \equiv \alpha + \delta(1-\phi) + \sigma_\pi\psi$, *and* $|\kappa| < 1$.

A.3.2 The impact of a policy parameter shift

We can now examine the derivative of b_n with respect to a shock to ϕ at time t (i.e., u_t^ϕ). This analysis quantifies the impact on bond yields of varying maturities when the central bank adjusts its response to inflation beyond market expectations.

First, note that the geometric series in Eq. (65) is:

$$\sum_{i=0}^{n-1}(\alpha + \delta(1-\phi) + \sigma_\pi\psi)^i = \frac{1-\kappa^n}{1-\kappa} \tag{77}$$

Next, we differentiate b_n with respect to u_t^ϕ to capture a shift in ϕ:

$$\frac{\partial b_n}{\partial u_t^\phi} = \frac{\sigma_\phi}{n}\frac{1-\kappa^n}{1-\kappa} + \frac{\phi}{n}\frac{(1-\kappa)(-n\kappa^{n-1}) - (1-\kappa^n)(-1)}{(1-\kappa)^2}(-\delta\sigma_\phi) \tag{78}$$

$$= \frac{\sigma_\phi}{n}\frac{1}{(1-\kappa)^2}[n\delta\phi\kappa^{n-1}(1-\kappa) - (1-\kappa^n)(\delta\phi+\kappa-1)]. \tag{79}$$

This derivative is positive for small values of n and negative for large values, under the assumption that

$$\alpha + \delta + \sigma_\pi\psi > 1, \tag{80}$$

a condition typically satisfied when inflation is persistent (i.e., α approaches 1) and $\psi > 0$. Therefore, a key prediction of the model is that if a central bank becomes more aggressive in responding to inflation, bond yields at short maturities increase, while those at longer maturities decrease, as in Smith and Taylor (2009). In Section A.4, we discuss the generalisation of these results to a setting where the central bank responds to both inflation and output gap.

It is interesting to note that for large n,

$$\frac{\partial b_n}{\partial u_t^\phi} \sim -\frac{\sigma_\phi}{n}\frac{1}{(1-\kappa)^2}(\delta\phi+\kappa-1). \tag{81}$$

Lemma 6 (Shock to the parameter of the policy rule). *If the parameter of the central bank's response to inflation changes, interest rates on bonds of sufficiently long maturity move in the opposite direction to the change in the parameter and the short-term rates.*

A.3.3 The impact of shocks to inflation

Let us first observe that the derivative of b_n in n is

$$\frac{\partial b_n}{\partial n} = \frac{\partial}{\partial n}\left(\frac{\phi}{n}\frac{1-\kappa^n}{1-\kappa}\right) = -\frac{1}{n}\frac{\phi}{1-\kappa}\left(\frac{1-\kappa^n}{n} + \kappa^n \log\kappa\right) < 0,$$

hence, the yield curve response coefficient to inflation is always positive (since $|b_n| > 0$ for all n) and decreases over the horizons, approaching zero at a rate of n^{-1}.

Lemma 7 (Response to inflation). *The yield curve response coefficient to inflation is always positive and decreases over the horizons, approaching zero at a rate of n^{-1}.*

A.3.4 Monetary policy shocks

The effect on the yield curve of a monetary policy shock is described by the coefficients c_n in Eq. (74), which can be rearranged as

$$c_n = \frac{\sigma_{mp}}{n}\left[\zeta^{n-1} + \kappa\frac{\kappa^{n-1} - \zeta^{n-1}}{\kappa - \zeta} + \frac{1-\zeta^{n-1}}{1-\zeta}\left(1 - \frac{\delta\phi}{1-\kappa}\right)\right], \tag{82}$$

Let first note that on impact ($n = 1$) the effect is positive and equal to $c_1 = \sigma_{mp}$, while after one period ($n = 2$) is equal to ζ. Since the first two terms in parentheses are positive for any n, the sign of c_n depends on the last term. If κ is sufficiently large, i.e.

$$\kappa > 1 - \delta\phi \qquad \Longrightarrow \qquad \alpha + \delta + \sigma_\pi\psi > 1,$$

then there will exist some n^* for which c_n becomes negative. Interestingly, it is the same condition that holds for b_n.

For large n, Eq. (74) behaves as

$$c_n \sim \frac{\sigma_{mp}}{n}\frac{1}{1-\zeta}\left(1 - \frac{\delta\phi}{1-\kappa}\right) = -\frac{\sigma_{mp}}{n}\frac{1}{1-\kappa}\frac{1}{1-\zeta}\left(\delta\phi + \kappa - 1\right), \tag{83}$$

which shows that the impact of the shock on the yield curve declines with $n^{-1}(1-\zeta)^{-1}$.

Lemma 8 (Monetary policy shock). *Following a monetary policy shock the interest rates on bonds of sufficiently long maturity will move in the opposite direction to the monetary policy shock and the movement of the short maturities. The magnitude of the effects declines with the maturity of the bond, at rate $n^{-1}(1-\zeta)^{-1}$.*

A.3.5 The magnitude of the effects at the end of the yield curve

Suppose $\zeta = 0$ and observe that:

$$c_n \sim -\frac{\sigma_{mp}}{n}\frac{1}{(1-\kappa)(1-\zeta)}\left(\delta\phi+\kappa-1\right) \tag{84}$$

$$\frac{\partial b_n}{\partial u_t^{\phi}} \sim -\frac{\sigma_{\phi}}{n}\frac{1}{(1-\kappa^2)}(\delta\phi+\kappa-1) \tag{85}$$

Hence conditionally on unit shocks (and unit variances), the long run effects of a shock to the rule as compared to a monetary policy shock are

$$\frac{1}{(1-\kappa)^2} \qquad \text{vs.} \qquad \frac{1}{1-\kappa}\frac{1}{1-\zeta},$$

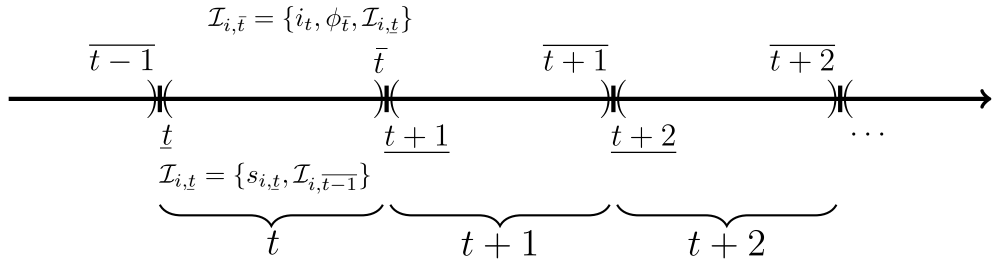
Thus, $\kappa = \alpha + \delta(1-\phi) + \sigma_{\pi}\psi > \zeta$, i.e. if the inflation persistence α is sufficiently larger than the autocorrelation of the monetary policy shocks, then the effects of a shock to the rule would impart a significant rotation to yield curve with its long end moving in opposite direction to the short end, while the monetary policy shock would have negligible effects on longer maturities.

Lemma 9 (**Magnitude of the effects at the end of the yield curve**)**.** *If the persistency of inflation is sufficiently higher than the autocorrelation of a monetary policy shocks, then at longer maturities, the impact of a monetary policy shock on yields is significantly smaller than the impact of a shift in policy rule parameters.*

A.3.6 Imperfect Information

We now embed the term structure model, in which the policy rule responds only to inflation, in an environment characterised by imperfect information, following Miranda-Agrippino and Ricco (2021) and Pirozhkova et al. (2024).

Figure A.2: The information flow



Notes: The figure presents the information flow in the economy. Agents do not observe directly the state of the economy but receive noisy private signals, $s_{i,\underline{t}}$ at the beginning of time t which are used to update their information set from the end of the previous period, i.e. $\mathcal{I}_{i,\overline{t-1}}$. The information set at the end of period t contains the observed policy rate and the signal about the new policy parameter, $\phi_{\bar{t}}$.

The inflation process in the model is:

$$\begin{aligned} \pi_t &= \alpha\pi_{t-1} - \delta(r_{t-1} - \pi_{t-1}) + \sigma_\pi u_t^\pi, \\ &= \alpha\pi_{t-1} - \delta((\phi + \widehat{\phi}_{t-1})\pi_{t-1} + \sigma_{mp}u_{t-1}^{mp} - \pi_{t-1}) + \sigma_\pi u_t^\pi, \\ &= (\alpha - \delta(\phi + \widehat{\phi}_{t-1} - 1))\pi_{t-1} - \delta\sigma_{mp}u_{t-1}^{mp} + \sigma_\pi u_t^\pi \end{aligned} \tag{86}$$

Each agent i in the economy do not directly observe π_t, but receives a private noisy signal of π_t at the beginning of the time period $t = [\underline{t}, \bar{t}]$ (see Figure A.2):

$$s_{i,\underline{t}} = \pi_t + \nu_{i,\underline{t}} \ , \qquad\qquad \nu_{i,\underline{t}} \sim \mathcal{N}(0, \sigma_{n,\nu}) \ . \tag{87}$$

Agent also form beliefs about the Taylor rule parameter, i.e. $\phi + \widehat{\phi}_{t-1}$, by assuming that it is equal to last period, i.e.

$$\phi_{\underline{t}} = \phi_{\overline{t-1}} = F_{\overline{t-1}}\left(\phi + \widehat{\phi}_{t-1}\right)$$

Given the signal, and conditional on their information set $\mathcal{I}_{\underline{t}} = \{s_{i,\underline{t}}, \phi_{\underline{t}}, \mathcal{I}_{\overline{t-1}}\}$, agents update their expectations from closing time of the previous period, $F_{i,\overline{t-1}}\pi_t$, and form expectations

$F_{i,\underline{t}}\pi_t$ given their information set via the Kalman filter

$$F_{i,\underline{t}}\pi_t = K_1 s_{i,\underline{t}} + (1-K_1)F_{i,\overline{t-1}}\pi_t \ , \tag{88}$$

$$F_{i,\underline{t}}\pi_{t+h} = (\alpha - \delta(\phi_{\underline{t}} - 1))^h F_{i,\underline{t}}\pi_t \qquad \forall h > 0 \ , \tag{89}$$

where K_1 is the Kalman gain which represent the relative weight placed on new information relative to previous forecasts. When the signal is perfectly revealing $K_1 = 1$, while in the presence of noise $K_1 < 1$. Thus $(1-K_1)$ is the degree of information rigidity faced by the agents.

Given their forecasts, at $\underline{t}$, agents trade bonds of different maturities with the following interest rates

$$i_{\underline{t}}^{(n)} = a_n + b_n F_{i,\underline{t}}\pi_t \ , \tag{90}$$

and prices

$$P_{\underline{t}}^{(n)} = e^{A_n + B_n F_{i,\underline{t}}\pi_t} \ . \tag{91}$$

At opening time $\underline{t}$ the central bank observes a private noisy signal of the state of the economy in period t

$$s_{cb,\underline{t}} = \pi_t + \nu_{cb,\underline{t}} \ , \qquad \nu_{cb,\underline{t}} \sim \mathcal{N}(0, \sigma_{cb,\nu}) \ . \tag{92}$$

We can assume without loss of generality that the signal observed by the central bank is more precise than the one observed by agents: $\sigma_{cb,\nu} < \sigma_{n,\nu}$. Given the signal, the central bank updates its expectations from closing time of the previous period given its information set via the Kalman filter

$$F_{cb,\underline{t}}\pi_t = K_{cb} s_{cb,\underline{t}} + (1-K_{cb})F_{cb,\overline{t-1}}\pi_t \ , \tag{93}$$

$$F_{cb,\underline{t}}\pi_{t+h} = (\alpha - \delta(\phi + \widehat{\phi}_t - 1))^h F_{cb,\underline{t}}\pi_t \qquad \forall h > 0 \ , \tag{94}$$

where K_{cb} is the bank's Kalman gain. Given its nowcast for inflation, the central bank sets and announces the interest rate, by following its policy rule:

$$i_t^{(1)} = r_t = (\phi + \widehat{\phi}_t)F_{cb,\underline{t}}\pi_t + \sigma_{mp}u_t^{mp} \tag{95}$$

At closing time $\bar{t}$, agents observe the new interest rate r_t and receive a noisy signal about the Taylor rule parameter of the central bank, i.e.

$$\phi_{\bar{t}} = \phi + \widehat{\phi}_t + \zeta_t \ , \qquad\qquad \zeta_t \sim \mathcal{N}(0, \sigma_\zeta) \ .$$

Given these two signals they update their expectations and trade bonds at different maturities.

The policy rate is to the agents a public signal about the state of the economy. In fact, the policy rate depends on the value of inflation at t as[9]

$$\begin{aligned}
r_t &= (\phi + \widehat{\phi}_t)F_{cb,\underline{t}}\pi_t + \sigma_{mp}u_t^{mp} \\
&= (\phi + \widehat{\phi}_t)(K_{cb}s_{cb,\underline{t}} + (1 - K_{cb})F_{cb,\overline{t-1}}\pi_t) + \sigma_{mp}u_t^{mp} \\
&= (\phi + \widehat{\phi}_t)(K_{cb}\pi_t + K_{cb}\nu_{cb,\underline{t}} + (1 - K_{cb})F_{cb,\overline{t-1}}\pi_t) + \sigma_{mp}u_t^{mp} \\
&= (\phi + \widehat{\phi}_t)(K_{cb}\pi_t + K_{cb}\nu_{cb,\underline{t}} + (1 - K_{cb})((\alpha - \delta(\phi - 1))F_{cb,\underline{t}}\pi_{t-1} - \delta\sigma_{mp}u_{t-1}^{mp}) + \sigma_{mp}u_t^{mp} \\
&= (\phi + \widehat{\phi}_t)\Bigg(K_{cb}\pi_t + K_{cb}\nu_{cb,\underline{t}} + \\
&\qquad (1 - K_{cb})\left(\left(\alpha - \delta\left(\phi + \widehat{\phi}_{t-1} - 1\right)\right)\left(\frac{r_{t-1} - \sigma_{mp}u_{t-1}^{mp}}{\phi + \widehat{\phi}_{t-1}}\right) - \delta\sigma_{mp}u_{t-1}^{mp}\right)\Bigg) + \sigma_{mp}u_t^{mp}.
\end{aligned}$$

Hence, conditionally, on observing r_t and r_{t-1} (and knowing K_{cb}), agents extract a public

[9] We assume that the central bank does not update its nowcast between $\underline{t}$ and $\bar{t}$, i.e. $F_{cb,\underline{t}}\pi_{t-1} = F_{cb,\bar{t}}\pi_{t-1}$.

signal on π_t[10]

$$
\begin{aligned}
\tilde{s}_t &= \pi_t + \nu_{cb,\underline{t}} - K_{cb}^{-1}(1-K_{cb})((\alpha - \delta(\phi_{\overline{t-1}} - 1))\phi_{\overline{t-1}}^{-1} - \delta))\sigma_{mp}u_{t-1}^{mp} - \phi_{\underline{t}}^{-1}K_{cb}^{-1}\sigma_{mp}u_t^{mp} \\
&= \pi_t + \widetilde{\nu}_{cb,\underline{t}} \qquad (96)
\end{aligned}
$$

At $\bar{t}$, conditional on this public signal, agents update their information set, $\mathcal{I}_{\bar{t}} = \{i_t, p_t, \mathcal{I}_{\underline{t}}\}$, and their forecasts

$$F_{i,\bar{t}}\pi_t = K_2\tilde{s}_{cb,\bar{t}} + (1-K_2)F_{i,\underline{t}}\pi_t \ , \qquad (97)$$

$$F_{i,\bar{t}}\pi_{t+1} = (\alpha+\delta)F_{i,\bar{t}}\pi_t - \delta r_t \ , \qquad (98)$$

$$F_{i,\bar{t}}\pi_{t+h} = (\alpha - \delta(\phi_{\underline{t}}^{-1} - 1))^{h-1}F_{i,\bar{t}}\pi_{t+1} \qquad \forall h > 1 \ , \qquad (99)$$

where K_2 is the Kalman gain, as given by the noise in the public signal $\tilde{\nu}_{cb,\underline{t}}$.

We can obtain an expression for the revision of expectations, from Eq. (97)

$$
\begin{aligned}
F_{i,\bar{t}}\pi_t - F_{i,\underline{t}}\pi_t &= K_2\left[\tilde{s}_{cb,\bar{t}} - F_{i,\underline{t}}\pi_t\right] \\
&= K_2(\pi_t + \tilde{\nu}_{cb,\bar{t}}) - K_2\left[K_1(\pi_t + \nu_{i,\underline{t}}) + (1-K_1)F_{i,\overline{t-1}}\pi_t\right] \\
&= K_2(1-K_1)\pi_t + K_2\tilde{\nu}_{cb,\bar{t}} - K_2K_1\nu_{i,\underline{t}} - K_2(1-K_1)F_{i,\overline{t-1}}\pi_t \\
&= K_2(1-K_1)\pi_t + K_2\tilde{\nu}_{cb,\bar{t}} - K_2K_1\nu_{i,\underline{t}} \\
&\qquad - K_2(1-K_1)((\alpha+\delta)F_{i,\overline{t-1}}\pi_{t-1} - \delta r_{t-1}) \\
&= K_2(1-K_1)((\alpha+\delta)\pi_{t-1} - \delta r_{t-1} + \sigma_\pi u_t^\pi) + K_2\tilde{\nu}_{cb,\bar{t}} - K_2K_1\nu_{i,\underline{t}} \\
&\qquad - K_2(1-K_1)((\alpha+\delta)F_{i,\overline{t-1}}\pi_{t-1} - \delta r_{t-1}) \\
&= K_2(1-K_1)(\alpha+\delta)\left[\pi_{t-1} - F_{i,\overline{t-1}}\pi_{t-1}\right] \\
&\qquad + K_2\left[(1-K_1)\sigma_\pi u_t^\pi + \tilde{\nu}_{cb,\bar{t}} - K_1\nu_{i,\underline{t}}\right]. \qquad (100)
\end{aligned}
$$

[10] The noise in the signal $\tilde{s}_t$ is not a white noise. Hence it does not fulfils the standard conditions under which the Kalman filter is derived. Unmodelled dynamics can degrade the filter performance. We abstract here from these consideration that would require agents to adopt more sophisticated filtering techniques.

The expression is obtained by employing in order (i) Eq. (96), (88), and (87); (ii) Eq. (98); and (iii) the dynamic equation for inflation $\pi_t = (\alpha+\delta)\pi_{t-1} - \delta r_{t-1} + \sigma_\pi u_t^\pi$. To find an expression for the forecast error $\left(\pi_{t-1} - F_{i,\overline{t-1}}\pi_{t-1}\right)$ in Eq. (100), first note that Eq. (97) implies

$$\pi_t - F_{i,\bar{t}}\pi_t = K_2^{-1}(1-K_2)\left(F_{i,\bar{t}}\pi_t - F_{i,\underline{t}}\pi_t\right) - \tilde{\nu}_{cb,\bar{t}}. \tag{101}$$

Then Eq. (101) one period earlier can be written as

$$\begin{aligned}
\pi_{t-1} - F_{i,\overline{t-1}}\pi_{t-1} &= K_2^{-1}(1-K_2)\left[F_{i,\overline{t-1}}\pi_{t-1} - F_{i,\underline{t-1}}\pi_{t-1}\right] - \tilde{\nu}_{cb,\overline{t-1}} \\
&= K_2^{-1}(1-K_2)(\alpha+\delta)^{-1}\left[(F_{i,\overline{t-1}}\pi_t - F_{i,\underline{t-1}}\pi_t)\right. \\
&\qquad \left. + \delta\left(r_{t-1} - F_{i,\underline{t-1}}r_{t-1}\right)\right] - \tilde{\nu}_{cb,\overline{t-1}}\ .
\end{aligned} \tag{102}$$

Substituting Eq. (102) into Eq. (100) yields

$$\begin{aligned}
F_{i,\bar{t}}\pi_t - F_{i,\underline{t}}\pi_t = (1-K_1)(1-K_2)&\left[(F_{i,\overline{t-1}}\pi_t - F_{i,\underline{t-1}}\pi_t) + \delta(r_{t-1} - F_{i,\underline{t-1}}r_{t-1})\right] \\
&+ K_2\left[(1-K_1)(\sigma_\pi u_t^\pi - (\alpha+\delta)\tilde{\nu}_{cb,\overline{t-1}}) + \tilde{\nu}_{cb,\bar{t}} - K_1\nu_{i,\underline{t}}\right].
\end{aligned} \tag{103}$$

Taking the average over the market, i.e. over the index i, one gets the following proposition.

Lemma 10 (Expectation revisions). *A policy announcement triggers a market-wide revision of expectations, i.e. the information effects, of the form*

$$\begin{aligned}
F_{\bar{t}}\pi_t - F_{\underline{t}}\pi_t = (1-K_1)&(1-K_2)(F_{\overline{t-1}}\pi_t - F_{\underline{t-1}}\pi_t) \\
&+ (1-K_1)(1-K_2)\delta(r_{t-1} - F_{\underline{t-1}}r_{t-1}) \\
&+ K_2\left[(1-K_1)\sigma_\pi u_t^\pi - (1-K_1)(\alpha+\delta)\tilde{\nu}_{cb,\overline{t-1}} + \tilde{\nu}_{cb,\bar{t}}\right].
\end{aligned} \tag{104}$$

From the point of view of the agents, it has to be true that

$$r_t = \phi_{\bar{t}}F_{i,\bar{t}}\pi_t + \sigma_{mp}F_{i,\bar{t}}\nu_t^{mp}, \tag{105}$$

i.e. the agents decompose the rate observed into their updated expectations of inflation and their guess of the value of the monetary policy shock, given their perceived parameter of the Taylor rule.

From Eq. (89) and (98), one can derive expression for the term $\phi(F_{\overline{t-1}}\pi_t - F_{\underline{t-1}}\pi_t)$ in Eq. (104):

$$\begin{aligned}
&\phi_{\underline{t-1}}(F_{\overline{t-1}}\pi_t - F_{\underline{t-1}}\pi_t) \\
&\qquad = \phi_{\underline{t-1}}(\alpha+\delta)F_{\overline{t-1}}\pi_{t-1} - \phi_{\underline{t-1}}\delta r_{t-1} - \phi_{\underline{t-1}}(\alpha - \delta\phi_{\underline{t-1}} + \delta)F_{\underline{t-1}}\pi_{t-1} \\
&\qquad = -(\phi_{\overline{t-1}} - \phi_{\underline{t-1}})(\alpha+\delta)F_{\overline{t-1}}\pi_{t-1} + (\alpha+\delta)(r_{t-1} - F_{\underline{t-1}}u^{mp}_{t-1}) \\
&\qquad\quad - \delta\phi_{\underline{t-1}}r_{t-1} - (\alpha - \delta\phi_{\underline{t-1}} + \delta)F_{\underline{t-1}}r_{t-1} \\
&\qquad = (\alpha+\delta-\phi_{\underline{t-1}}\delta)(r_{t-1} - F_{\underline{t-1}}r_{t-1}) + \\
&\qquad\quad -(\phi_{\overline{t-1}} - \phi_{\underline{t-1}})(\alpha+\delta)F_{\overline{t-1}}\pi_{t-1} - (\delta+\alpha)F_{\underline{t-1}}u^{mp}_{t-1}
\end{aligned}$$

We can substitute this expression in Eq. (103) to obtain

$$\begin{aligned}
r_t - F_{\underline{t}}r_t &= (\phi_{\bar{t}} - \phi_{\underline{t}})F_{\bar{t}}\pi_t + \sigma_{mp}F_{i,\bar{t}}u^{mp}_t \\
&\quad + (1-K_1)(1-K_2)\phi_{\underline{t}}\phi^{-1}_{\underline{t-1}}(\alpha+\delta-\phi_{\underline{t-1}}\delta)(r_{t-1} - F_{\underline{t-1}}r_{t-1}) \\
&\quad - \phi_{\underline{t}}\phi^{-1}_{\underline{t-1}}(\phi_{\overline{t-1}} - \phi_{\underline{t-1}})(\alpha+\delta)F_{\overline{t-1}}\pi_{t-1} - \phi_{\underline{t}}\phi^{-1}_{\underline{t-1}}(\delta+\alpha)F_{\underline{t-1}}u^{mp}_{t-1} \\
&\quad + \phi_{\underline{t}}(1-K_1)(1-K_2)\delta(r_{t-1} - F_{\underline{t-1}}r_{t-1}) \\
&\quad + \phi_{\underline{t}}K_2\left[(1-K_1)\sigma_\pi u^\pi_t - (1-K_1)(\alpha+\delta)\tilde{\nu}_{cb,\overline{t-1}} + \tilde{\nu}_{cb,\bar{t}}\right] \\
&= (1-K_1)(1-K_2)\left(\delta\phi_{\underline{t}} + \phi_{\underline{t}}\phi^{-1}_{\underline{t-1}}(\alpha+\delta-\phi_{\underline{t-1}}\delta)\right)(r_{t-1} - F_{\underline{t-1}}r_{t-1}) \\
&\quad + (\phi_{\bar{t}} - \phi_{\underline{t}})F_{\bar{t}}\pi_t - \phi_{\underline{t}}\phi^{-1}_{\underline{t-1}}(\phi_{\overline{t-1}} - \phi_{\underline{t-1}})(\alpha+\delta)F_{\overline{t-1}}\pi_{t-1} \\
&\quad + \sigma_{mp}F_{i,\bar{t}}u^{mp}_t - \phi_{\underline{t}}\phi^{-1}_{\underline{t-1}}(\delta+\alpha)F_{\underline{t-1}}u^{mp}_{t-1} \\
&\quad + \phi_{\underline{t}}K_2\left[(1-K_1)\sigma_\pi u^\pi_t - (1-K_1)(\alpha+\delta)\tilde{\nu}_{cb,\overline{t-1}} + \tilde{\nu}_{cb,\bar{t}}\right]
\end{aligned}$$

This equation shows that monetary policy surprises in a model with imperfect information are autocorrelated and depends on a convolution of past and current shocks.

We can rewrite the equation collecting the past monetary policy and observation noise terms (i.e. terms $\tilde{\nu}_{cb,\bar{t}}$) in υ_t:

$$\begin{aligned}r_t - F_{\underline{t}} r_t = (1-K_1)(1-K_2)\left(\phi_{\underline{t}}\phi_{\underline{t-1}}^{-1}(\alpha+\delta)\right)(r_{t-1} - F_{\underline{t-1}} r_{t-1}) \\ + (\phi_{\bar{t}} - \phi_{\underline{t}})F_{\bar{t}}\pi_t - \phi_{\underline{t}}\phi_{\underline{t-1}}^{-1}(\phi_{\overline{t-1}} - \phi_{\underline{t-1}})(\alpha+\delta)F_{\overline{t-1}}\pi_{t-1} \\ + \phi_{\underline{t}} K_2(1-K_1)\sigma_\pi u_t^\pi + \sigma_{mp} F_{i,\bar{t}} u_t^{mp} + \widetilde{\upsilon}_t\end{aligned}$$

Let us observe that

$$\begin{aligned}\phi_{\underline{t}}\phi_{\underline{t-1}}^{-1} &= \frac{\phi + \widehat{\phi}_{t-1} + \zeta_{t-1}}{\phi + \widehat{\phi}_{t-2} + \zeta_{t-2}} = \frac{\phi + \widehat{\phi}_{t-2} + \sigma_\phi u_{t-1}^\phi + \zeta_{t-1}}{\phi + \widehat{\phi}_{t-2} + \zeta_{t-2}} \\ &\approx 1 + \frac{\sigma_\phi}{\phi + \widehat{\phi}_{t-1}} u_{t-1}^\phi + \frac{1}{\phi + \widehat{\phi}_{t-1}}(\zeta_{t-1} - \zeta_{t-2}) \\ &\approx 1 + \frac{\sigma_\phi}{\phi} u_{t-1}^\phi + \frac{1}{\phi}(\zeta_{t-1} - \zeta_{t-2}), \qquad (106)\end{aligned}$$

and that

$$\begin{aligned}\phi_{\bar{t}} - \phi_{\underline{t}} &= \widehat{\phi}_t - \widehat{\phi}_{t-1} + \zeta_t - \zeta_{t-1} \\ &= \sigma_\phi u_t^\phi + \zeta_t - \zeta_{t-1}, \qquad (107)\end{aligned}$$

and putting the two results together, we note that

$$\begin{aligned}\phi_{\underline{t}}\phi_{\underline{t-1}}^{-1}(\phi_{\overline{t-1}} - \phi_{\underline{t-1}}) &\approx \left(1 + \frac{\sigma_\phi}{\phi} u_{t-1}^\phi + \frac{1}{\phi}(\zeta_{t-1} - \zeta_{t-2}),\right)\left(\sigma_\phi u_{t-1}^\phi + \zeta_{t-1} - \zeta_{t-2}\right) \\ &\approx \sigma_\phi u_{t-1}^\phi + \zeta_{t-1} - \zeta_{t-2} \qquad (108)\end{aligned}$$

We can now complete our derivations by using the law of motion of the Taylor rule parameter, drop all the third order terms (i.e. where a term of the type $u_t^\phi u_{t'}^\phi$ multiplies other shocks), and absorb all the terms in aggregate observational noise (i.e. in ζ_t and $\tilde{\nu}_{cb,\overline{t-1}}$) into υ_t. Thus we obtain the following lemma.

Lemma 11 (**Policy rate surprise**)**.** *The average market forecast error on the policy rate is*

$$
\begin{aligned}
r_t - F_{\underline{t}} r_t = {} & \underbrace{(1-K_1)(1-K_2)\,(\alpha+\delta)\,(r_{t-1} - F_{\underline{t-1}} r_{t-1})}_{\textit{autocorrelation}} \\
& + \underbrace{(1-K_1)(1-K_2)\,(\alpha+\delta)\,\phi^{-1}\sigma_\phi u^\phi_{t-1}(r_{t-1} - F_{\underline{t-1}} r_{t-1})}_{\textit{rule parameter change}} \\
& + \underbrace{\sigma_\phi u^\phi_t F_{\bar{t}}\pi_t - \sigma_\phi u^\phi_{t-1}(\alpha+\delta) F_{\overline{t-1}}\pi_{t-1} + \widehat{\phi}_{t-1} K_2(1-K_1)\sigma_\pi u^\pi_t}_{\textit{rule parameter change}} \\
& + \underbrace{\phi K_2(1-K_1)\sigma_\pi u^\pi_t}_{\textit{info effect}} + \sigma_{mp} F_{i,\bar{t}} u^{mp}_t + \upsilon_t.
\end{aligned} \tag{109}
$$

where υ_t is convolution of past and current shocks.

To understand the structure of surprises along the yield curve, we need to consider Eq. (90), and obtain the following proposition.

Lemma 12 (**Monetary policy surprises**)**.** *The price revisions, i.e., the monetary policy surprises, for bonds at longer maturities are*

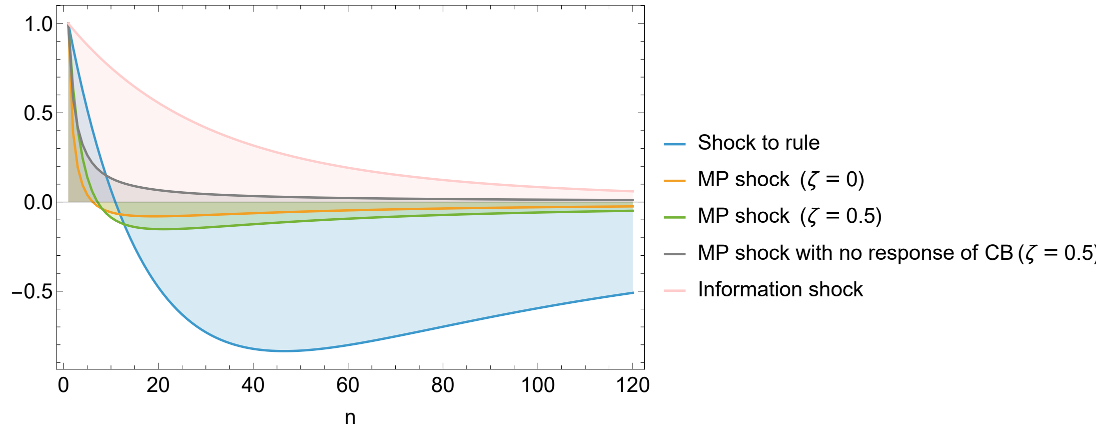
$$
\Delta_{[\bar{t}-\underline{t}]} i^{(n)}_t \equiv i^{(n)}_{\bar{t}} - i^{(n)}_{\underline{t}} = \underbrace{b_n (F_{\bar{t}}\pi_t - F_{\underline{t}}\pi_t)}_{\textit{info effect}} + \underbrace{\frac{\partial b_n}{\partial u^\phi_t}(\phi_{\bar{t}} - \phi_{\underline{t}})(F_{\bar{t}}\pi_t - F_{\underline{t}}\pi_t)}_{\textit{rule parameter change}} + \underbrace{c_n F_{\bar{t}} u^{mp}_t}_{\textit{mp shock}}\,, \tag{110}
$$

where the derivative of b_n with respect to u^ϕ_t, captures the shift in the policy parameter, and hence its effect on yield at different maturities.

We can now apply the insight provided by Lemmas 6, 7, 8 and 9 on the effects of shocks to the yield curve to the propagation of policy actions, under imperfect information. Figure A.3, provide a summary of the effects for different shocks, using the parametrisation proposed by Smith and Taylor (2009).

Lemma 13 (**Policy shocks, shocks to the rule and information**)**.** *Under imperfect information:*

Figure A.3: THE REACTION OF THE YIELD CURVE TO SHOCKS (MODEL II)



Notes: The figure compares the impact on the term structure of interest rates resulting from a shift in the policy rule (blue), a monetary policy shock (orange when $\zeta = 0$ and green when $\zeta = 0.5$), and an information shock (light red), for the model with endogenous term premium. The calibration, for quarterly data, follows Smith and Taylor (2009). $n = 120$ corresponds to 120 quarters (i.e., 30 years). Finally, in grey, the chart reports the term structure's reaction under the assumption that the central bank does not respond to the macroeconomic consequences of its own monetary policy shock, which follows an AR(1) process with an autocorrelation coefficient of 0.5.

(a) *A shock to the policy parameter, when understood by agents, causes interest rates on bonds with sufficiently long maturities to move in the opposite direction of both the parameter change and the short-term rate forecast error.*

(b) *An information shock raises the entire yield curve, with its effects diminishing over longer horizons.*

(c) *If inflation persistence significantly exceeds the autocorrelation of monetary policy shocks, a monetary policy shock raises short-term yields, exerts small negative effects on medium-term maturities, and has negligible effects on long-term maturities.*

A.4 Policy rule with inflation and output

Consider an extension of the model discussed in the previous section, where the central bank responds to real output alongside inflation:

$$r_t = \phi_\pi \pi_t + \phi_y y_t + \sigma_{mp}\nu_t^{mp} \;\; = \phi \mathbf{x}_t + \sigma_{mp}\nu_t^{mp} \tag{85}$$

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t' \lambda_t\right) \tag{111}$$

$$\lambda_t = \gamma + \Psi \mathbf{x}_t \tag{112}$$

$$y_t = \beta y_{t-1} - \delta(r_{t-1} - \pi_{t-1}) + \sigma_y u_t^y \tag{88}$$

$$\pi_t = \alpha \pi_{t-1} + \iota y_t + \sigma_\pi u_t^\pi \tag{113}$$

where we define

$$\mathbf{x}_t \equiv \begin{pmatrix} y_t \\ \pi_t \end{pmatrix}, \quad \mathbf{u}_t \equiv \begin{pmatrix} u_t^y \\ u_t^\pi \end{pmatrix}, \quad \boldsymbol{\phi} \equiv \begin{pmatrix} \phi_y \\ \phi_\pi \end{pmatrix}, \quad \boldsymbol{\gamma} \equiv \begin{pmatrix} \gamma_{01} \\ \gamma_{02} \end{pmatrix},$$

and

$$\Psi \equiv \begin{pmatrix} -\psi_{11} & \psi_{12} \\ \psi_{21} & -\psi_{22} \end{pmatrix}. \tag{114}$$

The shocks u_t^y, u_t^π, u_t^{mp} are i.i.d as $\mathcal{N}(0,1)$ and uncorrelated.

Equation (85) represents the policy rule, incorporating real output in the central bank's interest rate setting. Eq. (111) defines the pricing kernel, now extended to a matrix form. Eq. (112) describes risk terms related to inflation and real output, and Eq.s (88) and (113) model the dynamics of output and inflation.

The yield of an n-period bond remains an affine function of inflation and output, expressed as:

$$b_n^{(n)} = a_n + b_n' \mathbf{x}_t \tag{115}$$

where a_n is the intercept and b_n is the n-period response coefficient vector:

$$b_n = \begin{pmatrix} b_{1,n} \\ b_{2,n} \end{pmatrix}. \tag{116}$$

Rewriting Eq. (88) and (113) as a VAR:

$$\mathbf{x}_t = \Phi \mathbf{x}_{t-1} + \Sigma \mathbf{u}_t \tag{117}$$

where

$$\Omega = \begin{pmatrix} \beta - \delta\phi_y & -\delta(\phi_\pi - 1) \\ \iota(\beta - \delta\phi_y) & \alpha - \iota\delta(\phi_\pi - 1) \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_y & 0 \\ \iota\sigma_y & \sigma_\pi \end{pmatrix}. \tag{118}$$

As discussed in Smith and Taylor (2009), the response coefficient for bond yields is:

$$b_n = \frac{1}{n} \left(\sum_{i=0}^{n-1} (\Omega - \Sigma\Psi)^i \right)' \boldsymbol{\phi}, \tag{119}$$

which extends the previous formula in Eq. (66), and can be rewritten as

$$b_n = \frac{1}{n} \left[\left(I - (\Omega - \Sigma\Psi)^n\right) \left(I - (\Omega - \Sigma\Psi)\right)^{-1} \right]' \boldsymbol{\phi}. \tag{120}$$

and

$$\begin{aligned} b_n &\sim \frac{1}{n} \left[\left(I - (\Omega - \Sigma\Psi)\right)^{-1} \right]' \boldsymbol{\phi} \\ &= \frac{1}{n} \frac{1}{D} \begin{pmatrix} \iota(\delta(\phi_\pi - 1) + \psi_{12}\sigma_y) - \psi_{22}\sigma_\pi & \iota(\beta - \delta\phi_y + \psi_{11}\sigma_y) - \psi_{21}\sigma_\pi \\ \delta(1 - \phi_\pi) - \psi_{12}\sigma_y & 1 - \beta + \delta\phi_y - \psi_{11}\sigma_y \end{pmatrix} \boldsymbol{\phi}, \end{aligned} \tag{121}$$

where

$$D = (\beta - \delta\phi_y)\psi_{22}\sigma_\pi + \delta\iota(\phi_\pi - 1) + \iota\psi_{12}\sigma_y + \psi_{11}\psi_{22}\sigma_\pi\sigma_y - \psi_{12}\psi_{21}\sigma_\pi\sigma_y - \psi_{22}\sigma_\pi.$$

Define $\mathcal{K} \equiv \Phi - \Sigma\Psi$, for which

$$\kappa_{22} = \alpha - \iota\delta(\phi_\pi - 1) - \iota\sigma_y\psi_{12} + \sigma_\pi\psi_{22} = \alpha + \iota\kappa_{12} + \sigma_\pi\psi_{22}, \tag{122}$$

$$\kappa_{12} = -\delta(\phi_\pi - 1) - \sigma_y\psi_{12}, \tag{123}$$

$$\kappa_{11} = \beta - \delta\phi_y + \sigma_y\psi_{11}, \tag{124}$$

$$\kappa_{21} = \iota(\beta - \delta\phi_y) + \iota\sigma_y\psi_{11} - \sigma_\pi\psi_{21} = \iota\kappa_{11} - \sigma_\pi\psi_{21}. \tag{125}$$

The derivatives of b_n with respect to ϕ_y and ϕ_π are:

$$\frac{\partial b_n}{\partial \phi_y} = \frac{1}{n}\begin{pmatrix} 1 - \delta\sum_{i=1}^{n-1}\left(i\phi_y\kappa_{11}^{i-1} + \iota\phi_\pi\kappa_{21}^{i-1}\right) + \sum_{i=1}^{n-1}\kappa_{22}^{i} \\ \sum_{i=1}^{n-1}\kappa_{12}^{i} \end{pmatrix} \tag{126}$$

$$\frac{\partial b_n}{\partial \phi_\pi} = \frac{1}{n}\begin{pmatrix} \sum_{i=1}^{n-1}\kappa_{12}^{i} \\ 1 - \delta\sum_{i=1}^{n-1}\left(i\phi_y\kappa_{12}^{i-1} + \iota\phi_\pi\kappa_{22}^{i-1}\right) + \sum_{i=1}^{n-1}\kappa_{22}^{i} \end{pmatrix}. \tag{127}$$

This expression, is the equivalent of Eq. (79) in the model with the central bank responding only to inflation, and reflects counteracting effects. Two summations arise: one geometric, representing direct policy influence, and the other arithmetic-geometric, capturing inflation persistence and the output gap. Using this formulation, the policy response's effect on long-term yields can be assessed.

Let us start from the Eq. (127):

$$\frac{\partial b_{2,n}}{\partial \phi_\pi} = \frac{1}{n}\left(1 + \sum_{i=1}^{n-1}\kappa_{22}^{i-1}\right) - \frac{1}{n}\left(\delta\sum_{i=1}^{n-1} i(\phi_y\kappa_{12}^{i-1} + \iota\phi_\pi\kappa_{22}^{i-1})\right). \tag{128}$$

By computing the usual formulas for geometric and arithmetic series ones gets:

$$\frac{\partial b_{2,n}}{\partial \phi_\pi} = \frac{1}{n}\left(1 + \frac{1-\kappa_{22}^{n-1}}{1-\kappa_{22}} - \delta\left(\phi_y\frac{1 - n\kappa_{12}^{n-1} + (n-1)\kappa_{12}^{n}}{(1-\kappa_{12})^2} + \iota\phi_\pi\frac{1 - n\kappa_{22}^{n-1} + (n-1)\kappa_{22}^{n}}{(1-\kappa_{22})^2}\right)\right). \tag{129}$$

For n large we get:

$$\frac{\partial b_{2,n}}{\partial \phi_\pi} \sim \frac{1}{n}\left(1+\frac{1}{1-\kappa_{22}}-\delta\left(\phi_y\frac{1}{(1-\kappa_{12})^2}+\iota\phi_\pi\frac{1}{(1-\kappa_{22})^2}\right)\right).$$

Now consider the second entry in Eq. (127)

$$\frac{\partial b_{1,n}}{\partial \phi_\pi}=\sum_{i=1}^{n-1}\kappa_{21}^i=\kappa_{21}\frac{1-\kappa_{21}^{n-1}}{1-\kappa_{21}}. \tag{130}$$

Thus, for large n we get:

$$\frac{\partial b_{1,n}}{\partial \phi_\pi}=\frac{1}{n}\kappa_{21}\frac{1}{1-\kappa_{21}}. \tag{131}$$

Hence, the total effect on the yield curve of a change in ϕ_π is:

$$\begin{aligned}\frac{\partial b_n}{\partial \phi_\pi}=&\frac{\partial b_{1,n}}{\partial \phi_\pi}y_t+\frac{\partial b_{2,n}}{\partial \phi_\pi}\pi_t\\ =&\frac{1}{n}\left(\kappa_{21}\frac{1-\kappa_{21}^{n-1}}{1-\kappa_{21}}\right)y_t+\frac{1}{n}\left(1+\frac{1-\kappa_{22}^{n-1}}{1-\kappa_{22}}\right.\\ &\left.-\delta\left(\phi_y\frac{1-n\kappa_{12}^{n-1}+(n-1)\kappa_{12}^n}{(1-\kappa_{12})^2}+\iota\phi_\pi\frac{1-n\kappa_{22}^{n-1}+(n-1)\kappa_{22}^n}{(1-\kappa_{22})^2}\right)\right)\pi_t,\end{aligned} \tag{132}$$

and for large n is

$$\frac{\partial b_n}{\partial \phi_\pi}\sim\frac{\partial b_{1,n}}{\partial \phi_\pi}y_t+\frac{\partial b_{2,n}}{\partial \phi_\pi}\pi_t \tag{133}$$

$$=\frac{1}{n}\left(\frac{\kappa_{21}}{1-\kappa_{21}}\right)y_t+\frac{1}{n}\left(1+\frac{1}{1-\kappa_{22}}-\delta\left(\phi_y\frac{1}{(1-\kappa_{12})^2}+\iota\phi_\pi\frac{1}{(1-\kappa_{22})^2}\right)\right)\pi_t. \tag{134}$$

Consider now Eq. (126):

$$\frac{\partial b_{1,n}}{\partial \phi_y}=\frac{1}{n}\left(1+\sum_{i=1}^{n-1}\kappa_{22}^{i-1}\right)-\frac{1}{n}\left(\delta\sum_{i=1}^{n-1}i(\phi_y\kappa_{11}^{i-1}+\iota\phi_\pi\kappa_{21}^{i-1})\right), \tag{135}$$

and

$$\frac{\partial b_{2,n}}{\partial \phi_y}=\sum_{i=1}^{n-1}\kappa_{12}^i=\kappa_{12}\frac{1-\kappa_{12}^{n-1}}{1-\kappa_{12}}. \tag{136}$$

Thus, following the same steps:

$$\frac{\partial b_n}{\partial \phi_y} = \frac{\partial b_{1,n}}{\partial \phi_y} y_t + \frac{\partial b_{2,n}}{\partial \phi_y} \pi_t \tag{137}$$

$$\begin{aligned} &= \frac{1}{n}\left(\kappa_{12}\frac{1-\kappa_{12}^{n-1}}{1-\kappa_{12}}\right) y_t + \frac{1}{n}\left(1 + \frac{1-\kappa_{22}^{n-1}}{1-\kappa_{22}}\right. \\ &\left. - \delta\left(\phi_y \frac{1-n\kappa_{11}^{n-1}+(n-1)\kappa_{11}^{n}}{(1-\kappa_{11})^2} + \iota\phi_\pi \frac{1-n\kappa_{21}^{n-1}+(n-1)\kappa_{21}^{n}}{(1-\kappa_{21})^2}\right)\right)\pi_t, \end{aligned} \tag{138}$$

And for large n we get:

$$\frac{\partial b_n}{\partial \phi_y} \sim \frac{\partial b_{1,n}}{\partial \phi_y} y_t + \frac{\partial b_{2,n}}{\partial \phi_y} \pi_t \tag{139}$$

$$= \frac{1}{n}\left(\frac{\kappa_{12}}{1-\kappa_{12}}\right) y_t + \frac{1}{n}\left(1 + \frac{1}{1-\kappa_{22}} - \delta\left(\phi_y \frac{1}{(1-\kappa_{11})^2} + \iota\phi_\pi \frac{1}{(1-\kappa_{21})^2}\right)\right)\pi_t \tag{140}$$

Hence, for a shock of $\sigma_\phi u_t^\phi$, as in Eq.s (20-21) the total effect would be given by:

$$\frac{\partial i_t^{(n)}}{\partial \phi} = \frac{\partial b_{1,n}}{\partial \phi_y}\iota\sigma_\phi + \frac{\partial b_{2,n}}{\partial \phi_y}\sigma_\phi + \frac{\partial b_{1,n}}{\partial \phi_\pi}\iota\sigma_\phi + \frac{\partial b_{2,n}}{\partial \phi_\pi}\sigma_\phi \tag{141}$$

A.4.1 The magnitude of the effects at the long end of the curve

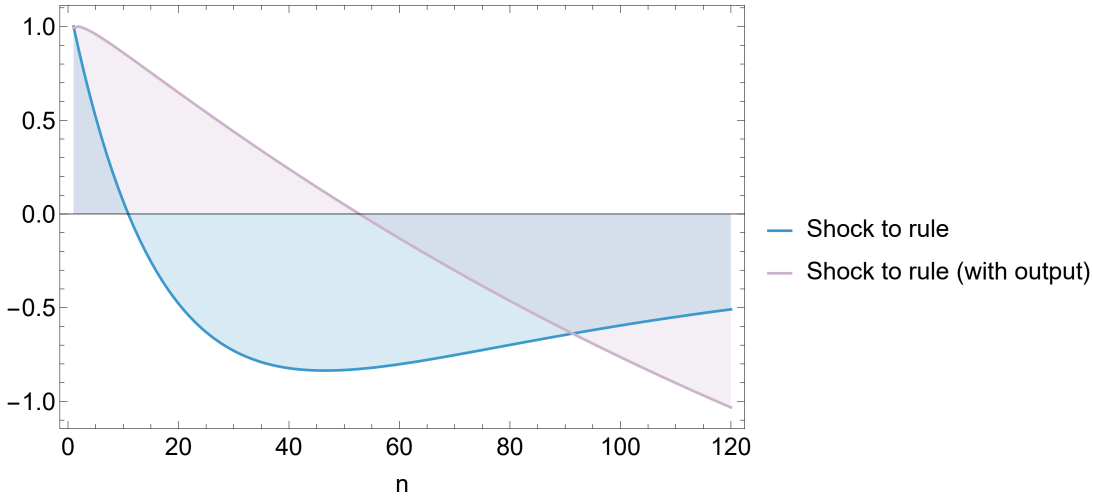
To assess the magnitude of the effects of a shock to the rule parameters, we use the parameter calibration of Smith and Taylor (2009). For large n, the dominant term in Eq. (141) determines the effects at longer maturities:[11]

$$\Delta_{[\bar{t}-\underline{t}]} i_t^{(n)} \sim \frac{1}{n}\left\{\frac{1}{1-\kappa_{22}}\right\}(\iota+1)\sigma_\phi - \frac{1}{n}\left\{\frac{\iota\delta\phi_\pi}{(1-\kappa_{22})^2}\right\}(\iota+1)\sigma_\phi.$$

It is important to observe that $\kappa_{22} = 0.946$, based on the parameter estimates of Smith and Taylor (2009). Since κ_{22} is close to one, for a reasonable range of values of the monetary policy autocorrelation coefficient, the shock to the rule induces a large effect on longer maturities,

[11] The calibration in Taylor and Smith is $\iota = 0.2$, $\sigma_y = 0.75$, $\delta = 0.2$, $\sigma_\pi = 0.36$, $\psi_{11} = 0.1$, $\psi_{12} = \psi_{21} = 0$, $\psi_{22} = 0.15$, $\beta = 0.2$. Assuming $\alpha = 0.95$, $\phi_\pi = 1.2$, and $\phi_y = 1.2$ we obtain: $\kappa_{12} = -0.04$, $\kappa_{11} = 0.035$, $\kappa_{21} = 0.007$, $\kappa_{22} = 0.946$.

Figure A.4: The reaction of the yield curve to shocks (model II with output)



Notes: The figure compares the impact on the term structure of interest rates resulting from a shift in the policy rule in the model without output in the full model of section A.3 (blue), with a shock to rule of the same model where we also model output, in violet. The calibration, for quarterly data, follows Smith and Taylor (2009), however we assume $\alpha = 0.95$ instead of $\alpha = 1$ (larger persistent of inflation) so that $|k_{22}| < 1$. $n = 120$ corresponds to a maturity of 30 years.

whereas the monetary policy shock has negligible effects. Figure A.4 plots the response of the yield curve to a shock to the rule in the model without output (in blue) versus the model with output (in violet). Notice that in both models, using the parameter estimates from Smith and Taylor (2009), the response at longer maturities becomes negative and large.

B Data sources

Table B.4: DATA

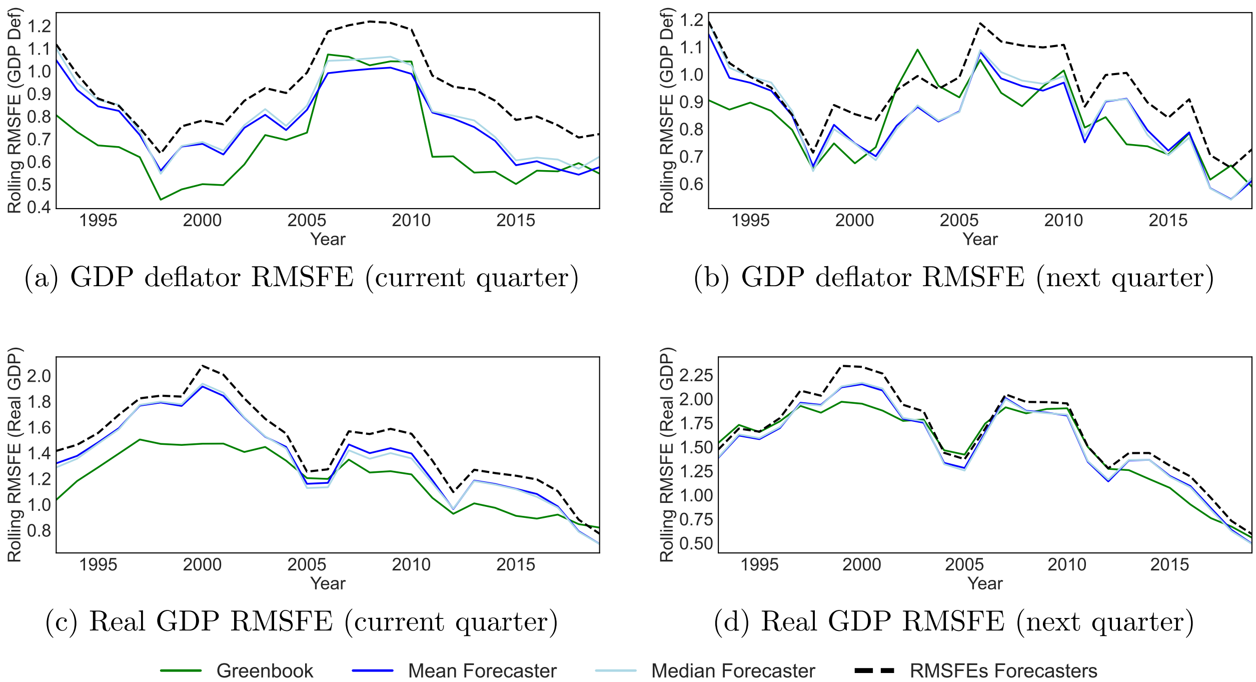
Figure	Data	Source
Figure (1)	• DGS2: Market Yield on U.S. Treasury Securities at 2-Year Constant Maturity, Monthly, Not Seasonally Adjusted, Percent • PCEPILFE: Personal Consumption Expenditures Excluding Food and Energy, Chain-Type Price Index, Index 2012=100, Monthly, Seasonally Adjusted • OUTGAP: Difference between GDP (GDPC1) and potential GDP (GDPPOT), expressed as a percentage of potential GDP, Monthly, Linearly Interpolated • GDPC1: Real Gross Domestic Product, Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate • GDPPOT: Real Potential Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Not Seasonally Adjusted	FRED
Figures (2) to (6)	• BCFFs for Federal Funds Rate, Percent per annum, average for quarter, 1993-2021 • BCFFs for Real GDP, Q/Q change, SAAR, 1993-2021 • BCFFs for GDP price index, Q/Q change, SAAR, 1993-2021 • 'First release' realisations for Real GNP/GDP (ROUTPUT), Q/Q Growth (Annual Rate, Percentage Points) • 'First release' realisations for Price Index for GNP/GDP (P), Q/Q Growth (Annual Rate, Percentage Points) • Effective Federal Funds rate	Blue Chip Financial Forecasts BEA & FRED
Figure (10)	• Dataset high-frequency price revisions	Gurkaynak et al. (2005)
Tables 3-4	• Greenbook projections for Q/Q growth in real GDP, chain weight (annualized percentage points) • Greenbook projections for Q/Q growth in price index for GDP, chain weight (annualized percentage points) • Greenbook projections for the unemployment rate (percentage points) • Note: data used also in Figures (2) to (6)	Tealbook (formerly Greenbook) Data Sets
Figures (12)-(14)	• INDPRO: IP Index • UNRATE: CivilianUnemploymentRate • FEDFUNDS: Effective Federal Funds Rate • GS1-GS5-GS10 Treasury Yields • CPIAUCSL: CPI :All Items • S&P500: S&P's Common Stock Price Index: Composite • Excess Bond Premium	FRED-MD & Gilchrist and Zakrajšek (2012)

Notes: Datasets adopted in the paper.

C Additional charts and figures

C.1 Remarks – additional charts

Figure C.5: Rolling RMSFE (Blue Chip same month Greenbook)



(a) GDP deflator RMSFE (current quarter)

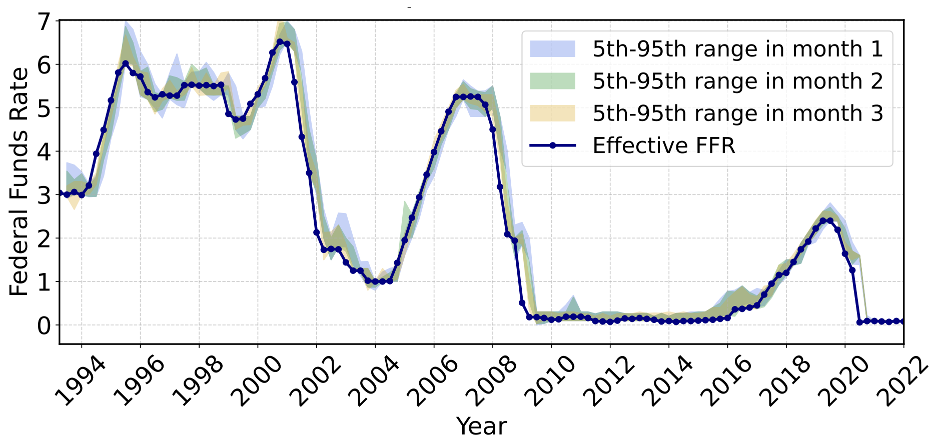
(b) GDP deflator RMSFE (next quarter)

(c) Real GDP RMSFE (current quarter)

(d) Real GDP RMSFE (next quarter)

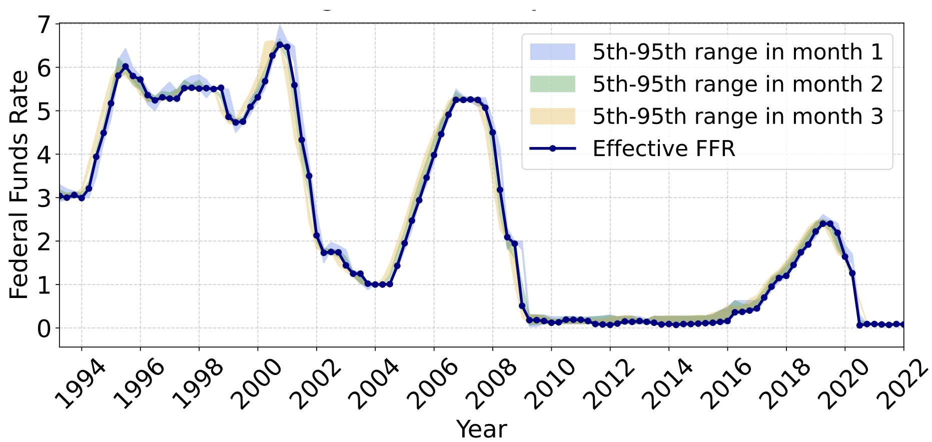
Notes: This figure shows rolling RMSFEs for quarter over quarter real GDP growth and quarter over quarter price deflator growth, for both the current quarter ($h = 0$) and the next quarter ($h = 1$). For each year, the RMSFEs are computed as 5-year centred moving averages of year over year averages of RMSFEs for each forecasters. The solid lines represent the rolling RMSFEs for the Greenbook (green), the mean forecaster (blue), and the median forecaster (light blue). The dashed black line corresponds to the average RMSFE across all individual forecasters. Greenbook forecasts are aligned with the closest preceding Blue Chip forecast date for each FOMC meeting. Only forecasters who have been consistently active for at least 15 years and provided current quarter forecasts in at least 6 months of each year are included in the computation of average RMSFEs. Results with a larger set of forecasters confirm better performance of the Greenbook with respect to the average RMSFEs across forecasters and similar performance with the other forecasters. Sample goes from 1993 to 2019 (end of availability of the Greenbook).

Figure C.6: Dispersion of one-quarter-ahead market forecasts for federal funds rate



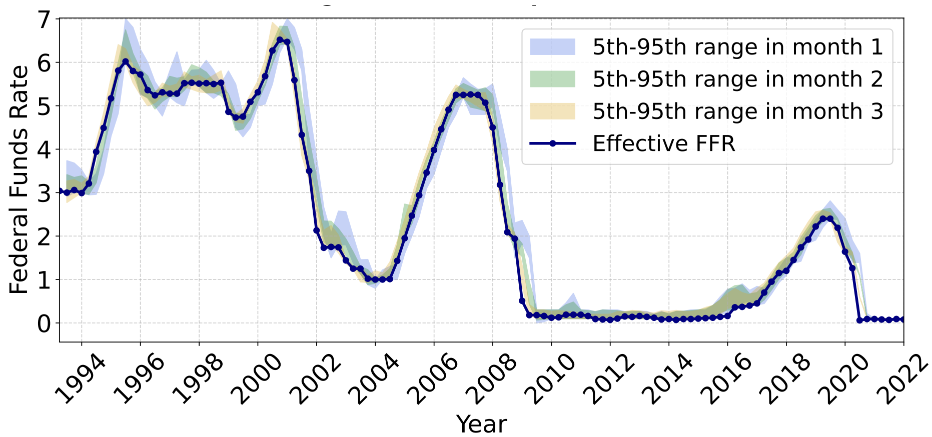
Notes: The figure illustrates the dispersion of market forecasts for the next quarter of the average FFR alongside the actual average realised rate. The purple area represents the range between the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentile of the forecasts done in the first month of each quarter. The green area represents the same range for the second month of each quarter, and the yellow area represents the same range for the last month of each quarter. The blue line corresponds to the realised quarterly average of the FFR.

Figure C.7: Dispersion of fixed-horizon market forecasts for federal funds rate (current quarter)



Notes: The figure illustrates the dispersion of fixed-horizon market forecasts for the current quarter of the average FFR, alongside the actual average realised rate. The purple area represents the range between the 5^{th} and 95^{th} percentile of the forecasts issued in the first month of each quarter. The green area represents the same range for the second month, and the yellow area for the third. The blue line corresponds to the realised quarterly average of the FFR.

Figure C.8: Dispersion of fixed-horizon market forecasts for federal funds rate (next quarter)



Notes: The figure illustrates the dispersion of fixed-horizon market forecasts for the next quarter of the average FFR, alongside the actual average realised rate. The purple area represents the range between the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentile of the forecasts issued in the first month of each quarter. The green area represents the same range for the second month, and the yellow area for the third. The blue line corresponds to the realised quarterly average of the FFR.

C.2 Comparison Greenbook and Consensus Economics

This section explains the procedure used to construct forecast evaluation figures for fixed-horizon predictions of inflation and real GDP growth for Greenbook projections and Consensus Economics. The forecasts come from two sources: private-sector projections compiled by the Consensus Economics, and internal Federal Reserve projections released in what was formerly known as the Greenbook, now referred to as the Tealbook. The aim is to compare these forecasts to realised inflation and real GDP growth over a consistent 12-month horizon.

Consensus forecasts are collected monthly and report the expected percentage change in both CPI and real GDP for the current calendar year and the next calendar year. Thus, these forecasts are provided as fixed-event forecasts fo the current year and next calendar year, but for evaluation purposes, we convert them to a fixed 12-month horizon beginning from the month of the survey. To achieve this, a weighted average of the current and next year's forecasts is computed. The weights are determined by the number of months remaining in the year of the forecast. For instance, if the survey is conducted in April, then 9 months remain in the current year, resulting in weights of 9/12 for the current year's forecast and 3/12 for the next year's. This method produces a continuous rolling estimate of the expected 12-month change in CPI and GDP, aligning all consensus forecasts to the same horizon for comparability.

The Greenbook projections differ in both structure and format. These forecasts are reported quarterly and expressed as annualised quarter-on-quarter growth rates. In order to construct a 12-month forecast comparable to that derived from the Consensus data, a compound transformation is applied. Specifically, the forecasted growth rates for the next four quarters are first converted from annualised percentages to decimal quarterly rates. Then, a compounded growth factor over the four quarters is calculated by multiplying the one-plus-growth terms together and subtracting one. The result reflects the expected year-on-year change from the time of the forecast, and the same transformation is applied to both real GDP and CPI projections from the Greenbook.

After transforming all forecasts into a common 12-month format, they are matched to observed realisations. For inflation, the realised value is based on the Consumer Price Index for All Urban Consumers (CPI-All Items), which is publicly available from the FRED under the series 'CPIAUCSL'. To align realisations with the forecast horizon, the actual CPI 12-month growth rate is computed by taking the percentage change in the CPI index from the survey month to the same month one year later. The resulting figure is a year-ahead realisation that directly matches the intended forecast horizon.

The realised values for real GDP growth are constructed directly from first-release quarterly data available in the Philadelphia Fed's Real-Time Data. This dataset reports annualised quarter-on-quarter growth rates, which must be transformed into year-on-year changes to match the forecast horizon. To do this, each quarterly growth rate is first converted to a decimal rate by dividing by 100 and then by 4 (since it is reported as an annualised quarterly rate). Then, a forward-shifted rolling product of four consecutive quarters is taken to simulate the compounded effect over the upcoming year. The formula mirrors the transformation used for the Greenbook forecasts. The result is a series of forward-looking, realised 12-month growth rates that are aligned with each survey date.

Forecast performance is assessed using the RMSFE, which is calculated by taking the square root of the mean of squared forecast errors. RMSFEs are computed separately for each individual forecaster, as well as for the mean and median of the forecasts reported by the Consensus Economics panel, and for the Greenbook forecast.

Two additional benchmarks are constructed for comparison. The first is the mean of individual RMSFEs across all forecasters who meet a consistency threshold, defined as having submitted forecasts in at least six months of a year, and in at least ten different years. This average reflects typical individual performance within the forecasting panel. The second benchmark is the pooled RMSFE, which is computed by collecting all raw individual monthly forecast errors, squaring them, averaging, and taking the square root. This measure treats the entire panel's forecast distribution as a single group and reflects the total collective forecasting accuracy.

To ensure consistency in the evaluation of forecast performance, all forecasts and realisations are aligned using the Greenbook projection dates as reference points. We match the most recent previous available forecast for each private forecaster with respect to each Greenbook date. Finally, for each Greenbook release, the corresponding realised value, be it inflation or real GDP growth, is matched to the 12-month change beginning from the date of that projection.

These metrics are summarised in bar plots, as shown in Figures (C.9a) and (C.9b), which display full-sample (1990 to 2019) RMSFEs for inflation and real GDP growth, respectively. The bars indicate RMSFEs for each consistent individual forecaster. Vertical lines denote the pooled RMSFE (black dashed line), the mean forecast (blue), the median forecast (light blue), and the Greenbook (solid green line).

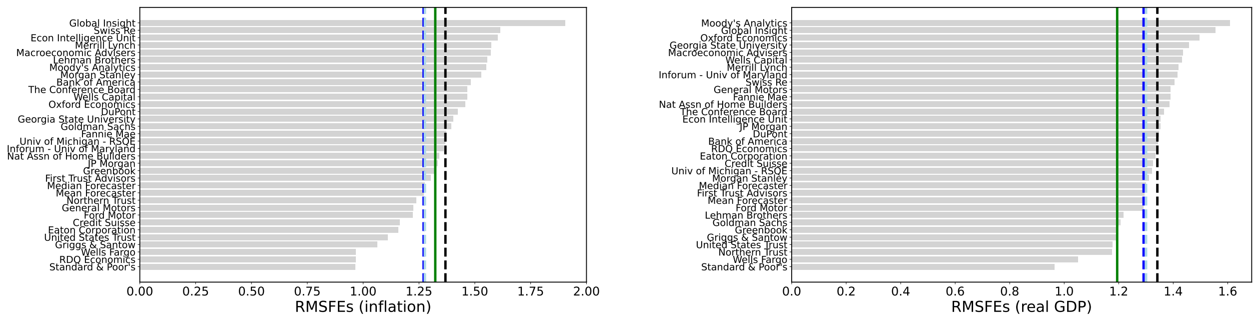
To evaluate how forecast accuracy changes over time, a rolling RMSFE analysis is conducted. This process begins by computing one-year-ahead RMSFEs at the annual level. For each calendar year, all forecast errors from that year's surveys are gathered for each forecast type—Greenbook, mean, median, and each individual forecaster.

After obtaining these annual RMSFEs, a centred five-year rolling average is applied. For each rolling year t, the RMSFE is averaged over the five-year window from $t-2$ to $t+2$. This is done identically for the Greenbook, mean, and median forecast series.

The pooled RMSFE in the rolling framework follows the same logic. For each year, all monthly forecast errors across all consistent individual forecasters are gathered. These are squared, averaged, and square-rooted to yield an annual pooled RMSFE. The resulting pooled series is then smoothed using the same five-year rolling window.

These rolling RMSFE series are visualised in Figures (C.10a) and (C.10b). Each line in the figure corresponds to Greenbook (green line), mean forecast (blue line), median forecast (light blue line), and the pooled RMSFEs (black line). All series are constructed by first computing annual RMSFEs for 1-year-ahead forecast errors, then applying a five-year centred moving average.

Figure C.9: RMSFE comparisons – Consensus Economics and Greenbook

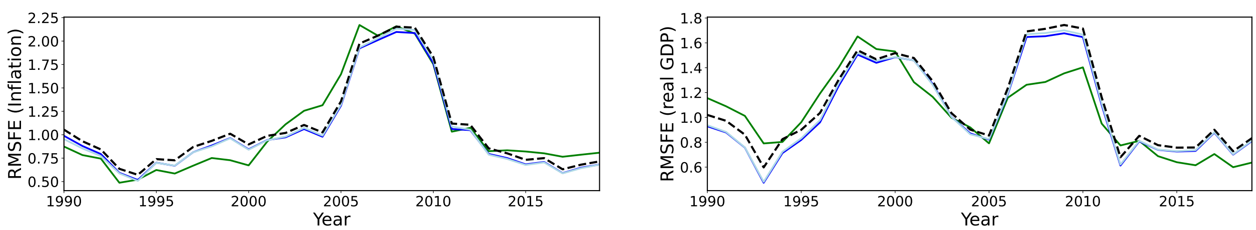


(a) Inflation RMSFEs (1990 to 2019)

(b) Real GDP RMSFEs (1990 to 2019)

Notes: Bars represent full-sample (1990 to 2019) RMSFEs for each forecaster in one year ahead forecasts error for growth in real GDP and inflation. Vertical lines mark RMSFEs for Greenbook (green), mean forecast (blue), median forecast (light blue), and pooled RMSFEs (black dashed). For any additional details, please refer to the text.

Figure C.10: Rolling RMSFEs – Consensus Economics and Greenbook



(a) Rolling inflation RMSFEs

(b) Rolling Real GDP RMSFEs

Notes: Each series is computed from annual RMSFEs of one-year-ahead forecast errors and the values shown are centred five-year averages of each series. The green line represents Greenbook, the blue line is the mean forecaster, the light blue line is the median forecaster, and the black line represents the pooled RMSFEs. For any additional details, please refer to the text.

C.3 Comparison Greenbook and BCFF

This section describes the methodology used to evaluate and compare the forecast accuracy of the Greenbook projections and private-sector forecasts from the BCFF. The analysis focuses on quarter-on-quarter annualised percentage changes in real GDP and the GDP deflator, evaluated at two fixed horizons: the current quarter ($h = 0$) and the next quarter ($h = 1$). Realised outcomes are based on first-release data obtained from the Philadelphia Fed's real-time data archive and correspond to the same quarterly periods targeted by the forecasts.

The forecast evaluation relies on a balanced panel of professional forecasters from the BCFF survey, which has been published monthly since 1980.[12] To ensure comparability over time and avoid distortions due to irregular participation, we restrict attention to a set of consistent forecasters defined as those who report forecasts for at least 15 years and submit projections in at least six months of each year at the $h = 0$ horizon. These criteria filter out occasional contributors and ensure that our performance metrics reflect sustained forecasting behaviour.

Greenbook forecasts are released five years after each FOMC meeting they refer to but are made shortly before those meetings and provide projections for different macroeconomic variables. For the purposes of this analysis, we focus exclusively on real GDP growth and GDP deflator, reported as annualised quarter-on-quarter rates. Two forecast horizons are evaluated: the current quarter and the following quarter. The corresponding realisations are taken to be the first official releases of real GDP and GDP deflator from the Philadelphia Fed's real-time data archive for the target quarter implied by each Greenbook forecast date.[13]

To allow for a meaningful comparison between Greenbook and BCFF projections, forecast dates must be aligned. Because the Greenbook is tied to the FOMC calendar and the BCFF is

[12] We start the analysis from 1993 as the label for forecasters become available from that date and helps us following individual forecasters over time.

[13] There are different releases of QoQ growth in real GDP and GDP deflator. We focus on the first release as it is the closest in time to when the forecasts are made. Results are similar when using the second and third releases, as also shown in Hoesch et al. (2023).

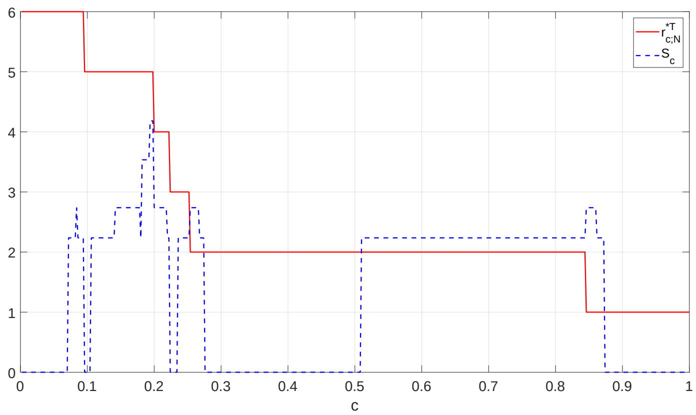
released on a monthly schedule, we implement two matching procedures. Under the 'previous date' alignment, we assign to each Greenbook release the most recent BCFF forecast that precedes it in calendar time. In contrast, following Bauer and Swanson (2023a), we consider also the 'closest date' approach that selects the Blue Chip forecast with the smallest number of days separating it from the Greenbook date, whether before or after. The BCFF are realeased on the first day of the month, and we take that date as the BCFF reference date. Also, to avoid cases where forecasts refer to different quarters (e.g., Greenbook in March matched with BCFF in April), we exclude all observations for which the Greenbook date falls in March, June, September, or December. These months correspond to ends of quarters, when a subsequent Blue Chip forecast might refer to a different reference period.

Forecast accuracy is assessed using the RMSFE. For each forecast type (Greenbook, mean forecaster, median forecaster, and individual forecasters) RMSFEs are calculated over the full sample from 1993 to 2019. The results are presented in static bar plots that show individual RMSFEs for each consistent forecaster, alongside vertical lines indicating the benchmark performances of the Greenbook, the mean forecaster, and the median forecaster. These static comparisons are shown separately for the 'closest date' and 'previous date' alignments in Figures (2) and (3), respectively.

To explore the evolution of forecast accuracy over time, we construct rolling RMSFEs. First, monthly forecast errors are aggregated at the annual level to produce year-specific RMSFEs for each forecaster. A centred five-year moving average is then applied to each series: for year t, the average is taken over the window $[t-2, t+2]$. This rolling procedure is applied separately to the Greenbook, the mean and median forecasters, and to the annual average across RMSFEs of individual forecasters. The resulting smoothed RMSFE series are plotted over time in Figure (4), allowing for a dynamic comparison of forecast performance. All computations are based exclusively on forecasts and realisations expressed as annualised quarter-on-quarter growth rates, ensuring comparability across forecasts and realisations.

C.4 Additional charts and tables for information effects

Figure C.11: Alessi et al. (2010) test for the number of factors



Notes: The figure reports the test proposed by Alessi et al. (2010). It plots $r^{*T}_{c,N}$ as a function of the parameter c, which represents the penalisation term for the information criterion used to determine the number of factors. The second stability interval, where S_c equals zero, corresponds to $r^{*T}_{c,N} = 4$, indicating the presence of four statistically significant factors.

Table C.11: Information effects on market surprises I (OLS post LASSO)

	FF1	FF2	FF3	FF4	FF5	FF6	ED1	ED2	ED3	ED4
$\text{RGDP}_{h=0}$	0.008*	0.005	0.004	0.007*	0.008*	0.011***	0.006	0.010***	0.009*	0.008*
	(0.003)	(0.004)	(0.003)	(0.004)	(0.003)	(0.003)	(0.004)	(0.004)	(0.005)	(0.005)
$\text{RGDP}_{h=3}$	-0.005									
	(0.004)									
$\text{PGDP}_{h=1}$	-0.006	-0.009*								
	(0.004)	(0.006)								
$\text{UNEMP}_{h=0}$	0.003									
	(0.002)									
$\Delta\text{UNEMP}_{h=2}$	0.027***									
	(0.017)									
$\text{RGDP}_{h=1}$		0.003	0.006	0.002			0.007	0.002	0.006	0.005
		(0.005)	(0.005)	(0.005)			(0.006)	(0.006)	(0.007)	(0.007)
$\text{PGDP}_{h=3}$				-0.010***						
				(0.008)						
$\Delta\text{RGDP}_{h=-1}$				-0.006**						
				(0.003)						
$\Delta\text{RGDP}_{h=1}$				0.008*			0.012***	0.010***	0.010***	
				(0.007)			(0.008)	(0.009)	(0.009)	
$\Delta\text{PGDP}_{h=0}$							0.010***			
							(0.008)			
$\Delta\text{RGDP}_{h=0}$								0.006	0.007*	
								(0.006)	(0.006)	
Constant	-0.017***	-0.011***	-0.032***	-0.010***	-0.030***	-0.037***	-0.041***	-0.040***	-0.045***	-0.042***
	(0.014)	(0.017)	(0.011)	(0.018)	(0.009)	(0.009)	(0.014)	(0.013)	(0.014)	(0.014)
$\mathcal{R}^2$	0.078	0.058	0.060	0.087	0.051	0.070	0.089	0.136	0.148	0.130
F	2.699	2.940	4.278	3.305	8.220	13.575	4.624	4.713	5.669	4.850
P-value	0.022	0.034	0.015	0.007	0.005	0.000	0.011	0.001	0.000	0.001
N	243	243	243	243	243	243	243	243	243	243

Notes: This table presents the results of the information effect regressions for short-term interest rates (FF1-FF6) and Eurodollar futures (ED1-ED4). The dependent variables are the monetary surprises, while the explanatory variables include Greenbook forecasts. The regressors have been selected by LASSO and the table presents OLS regressions post LASSO. Sample goes from 1991m7 to 2019m6.

Table C.11: Information effects on market surprises II (OLS post LASSO)

	TRE3M	TRE6M	TRE2	TRE5	TRE10	TRE30	SP500
$RGDP_{h=0}$	0.005**	0.006***	0.008**	0.009***	0.005	0.004	
	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.003)	
$PGDP_{h=1}$	-0.004						
	(0.005)						
$UNEMP_{h=0}$	0.002*						
	(0.001)						
$\Delta UNEMP_{h=2}$	0.033*						
	(0.017)						
$RGDP_{h=1}$			0.002				
			(0.005)				
$\Delta RGDP_{h=1}$			0.010	0.007			
			(0.007)	(0.008)			
$\Delta RGDP_{h=2}$				0.018*	0.029**	0.026***	
				(0.011)	(0.013)	(0.009)	
$RGDP_{h=-1}$					0.003	0.001	
					(0.002)	(0.002)	
$RGDP_{h=3}$					0.004	0.004	
					(0.004)	(0.003)	
$PGDP_{h=-1}$					0.008**	0.005	
					(0.004)	(0.003)	
$\Delta RGDP_{h=0}$					0.007		
					(0.005)		
Constant	-0.025	-0.022***	-0.028**	-0.022***	-0.046**	-0.031*	0.014
	(0.017)	(0.006)	(0.011)	(0.008)	(0.020)	(0.016)	(0.037)
$\mathcal{R}^2$	0.051	0.041	0.099	0.127	0.251	0.194	0.000
F	2.596	8.017	7.348	6.177	2.233	2.278	0.000
P-value	0.037	0.005	0.000	0.000	0.021	0.029	.
N	243	243	243	243	243	243	243

Notes: This table presents the results of the information effect regressions for treasury yields (TRE3M-TRE30Y) and Stock Market (SP500). The dependent variables are the monetary surprises, while the explanatory variables include Greenbook forecasts selected by LASSO. The table presents OLS regression post LASSO. Sample goes from 1991m7 to 2019m6 (availability of Greenbook).

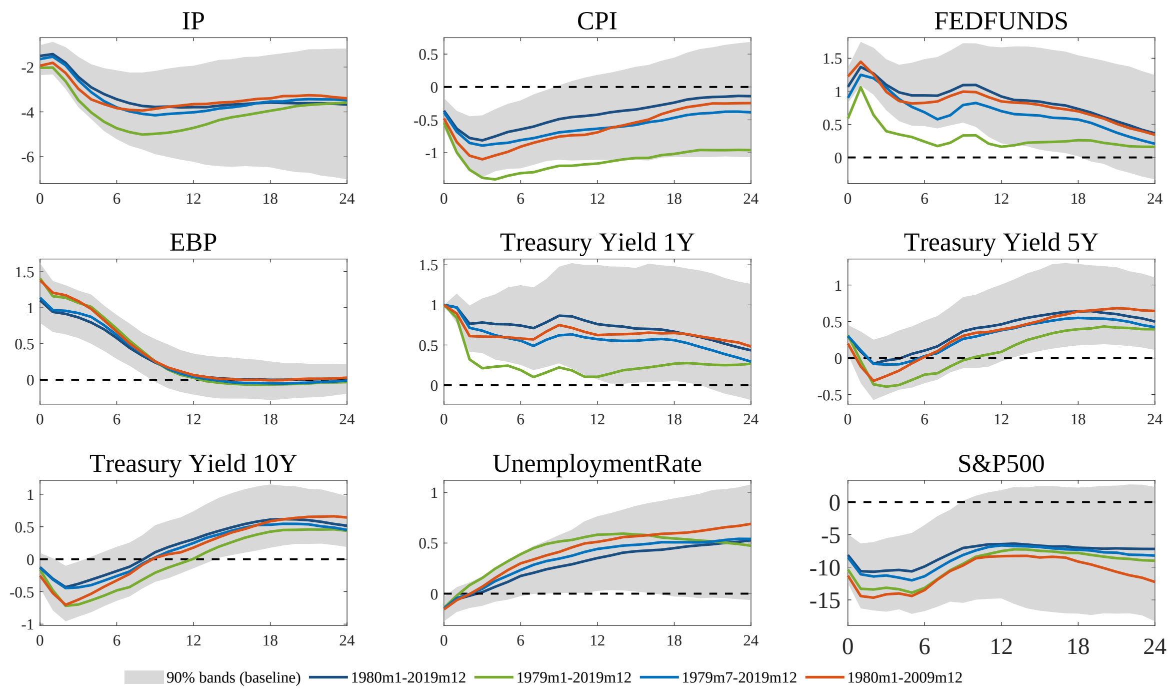
Table C.11: INFORMATION EFFECTS ON RAW IDENTIFIED MONETARY POLICY FACTORS

	Conventional MP (F1)	Shock to Rule (F2)	QE/QT (F3)	Forward Guidance (F4)
$\mathrm{RGDP}_{h=0}$	0.073		0.026	
	(0.073)		(0.068)	
$\mathrm{RGDP}_{h=1}$	0.054			
	(0.079)			
$\Delta\mathrm{RGDP}_{h=1}$		-0.083*		
		(0.043)		
Δ $\mathrm{RGDP}_{h=2}$		-0.146***	0.304	-0.019
		(0.051)	(0.219)	(0.017)
$\mathrm{RGDP}_{h=-1}$			0.068*	
			(0.035)	
$\mathrm{RGDP}_{h=3}$			0.066	
			(0.075)	
$\mathrm{PGDP}_{h=-1}$			0.118*	
			(0.067)	
Δ $\mathrm{gRGDP}_{h=0}$			0.123	0.016
			(0.084)	(0.011)
$\mathrm{gPGDP}_{h=0}$				0.008
				(0.008)
$\mathrm{gPGDP}_{h=3}$				0.028
				(0.025)
$\mathrm{UNEMP}_{h=2}$				0.005
				(0.005)
$\Delta\mathrm{RGDP}_{h=-1}$				0.010
				(0.009)
$\Delta\mathrm{PGDP}_{h=-1}$				-0.012
				(0.014)
$\Delta\mathrm{PGDP}_{h=1}$				-0.022
				(0.017)
Δ $\mathrm{UNEMP}_{h=-1}$				-0.173
				(0.115)
$\Delta\mathrm{UNEMP}_{h=2}$				0.102
				(0.072)
Constant	-0.295*	-0.014	-0.605*	-0.102
	(0.161)	(0.020)	(0.342)	(0.063)
$\mathcal{R}^2$	0.041	0.091	0.113	0.052
F-statistic	2.813	10.793	1.378	1.356
P-value	0.062	0.000	0.224	0.202
N	243	243	243	243

Notes: This table presents the information effect regressions for the raw monetary policy factors. The dependent variables are monetary policy factors (F1-F4), while the regressors include Greenbook forecasts. Sample spans from 1991m7 to 2019m6.

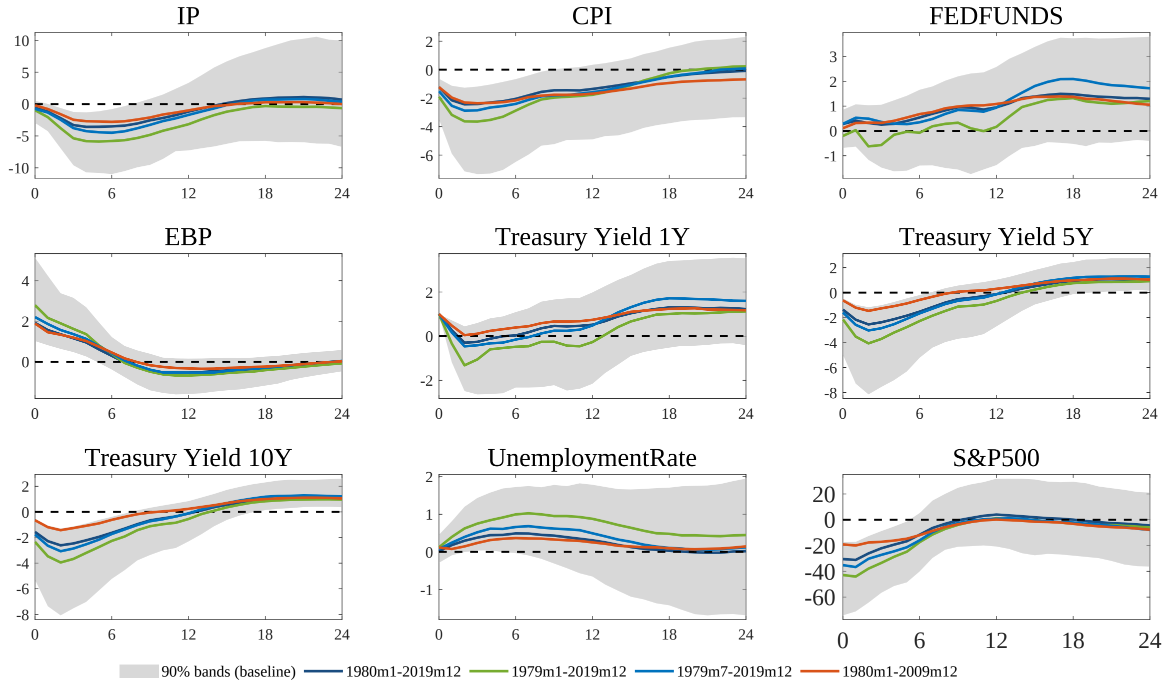
C.5 Robustness – IRFs different samples

Figure C.12: Conventional monetary policy – different samples



Notes: The figure reports the impulse response functions (IRFs) to a conventional monetary policy shock estimated over different starting periods of the sample. We also report, in orange, the results if we run the BVAR up to the financial crisis. The shock is identified with the conventional mp informationally robust factor, and normalised to induce a 100 basis point increase in the 1-Year Treasury rate. The grey shaded areas represent 90 percent coverage bands from the baseline specification.

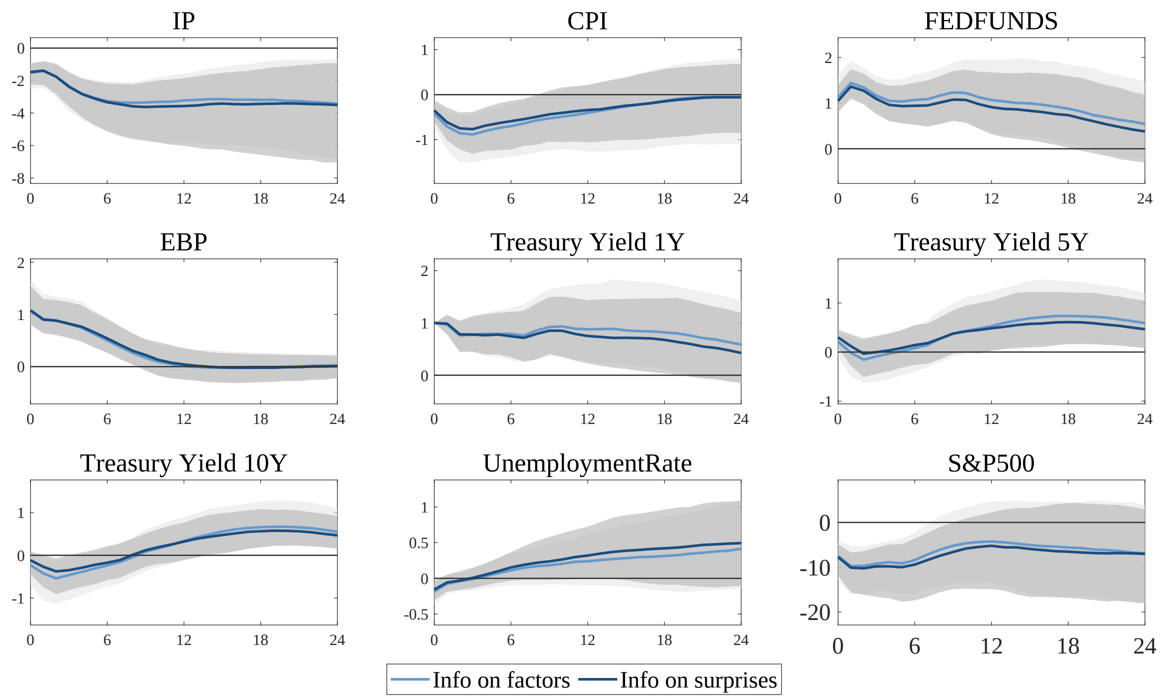
Figure C.13: Shock to the rule's parameters – different samples (information correction)



Notes: This figure presents the IRFs for a shock to the rule's parameters, over different starting periods of the sample. We also report, in orange, the results if we run the BVAR up to the financial crisis. The shock is normalised to induce a 100 basis point increase in the 1-Year Treasury rate. Grey shaded areas indicate 90 percent coverage bands from the baseline specification.

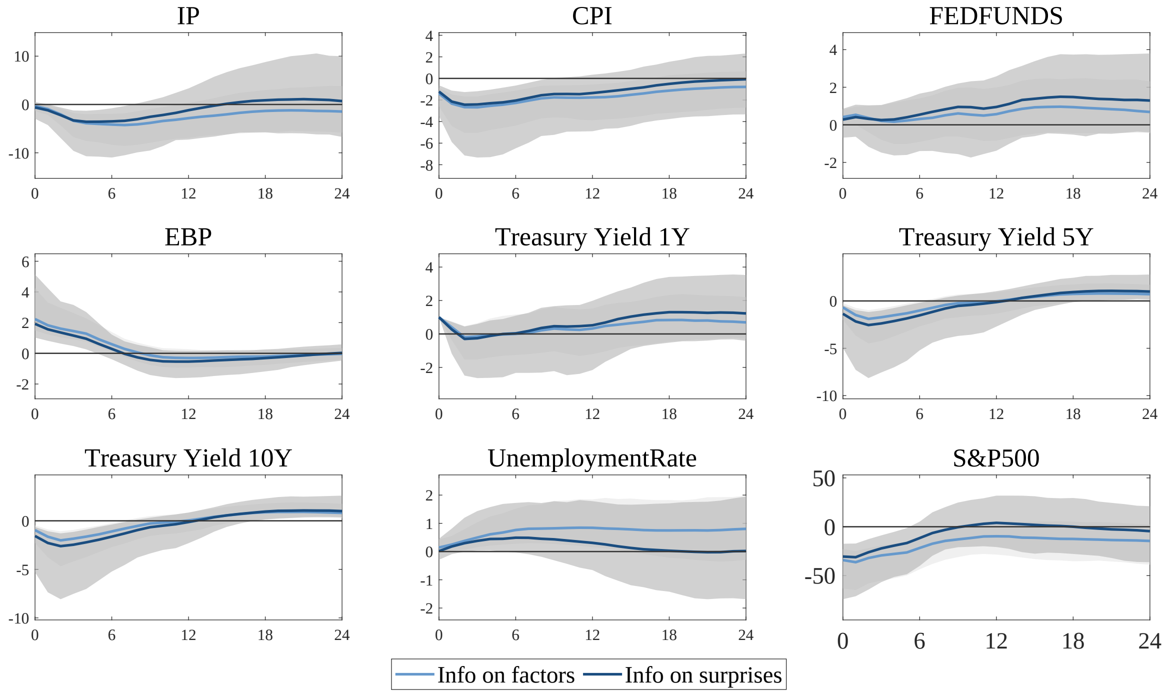
C.6 Comparison of different information corrections

Figure C.14: Comparison of information corrections conventional MP shock



Notes: The figure compares the IRFs for a conventional monetary policy shock using two different methods for information correction. The dark blue line corresponds to the correction applied to market surprises, while the light blue line represents the correction applied directly to the extracted factors.

Figure C.15: Comparison of information corrections shock to the rule's parameters



Notes: The figure compares the IRFs for a shock to the rule's parameters using two different information correction methods. The dark blue line represents the correction applied to market surprises, while the light blue line represents the correction applied to the extracted factors.

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